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ENVIRONMENTAL AND WATER RESOURCE CONSULTANTS

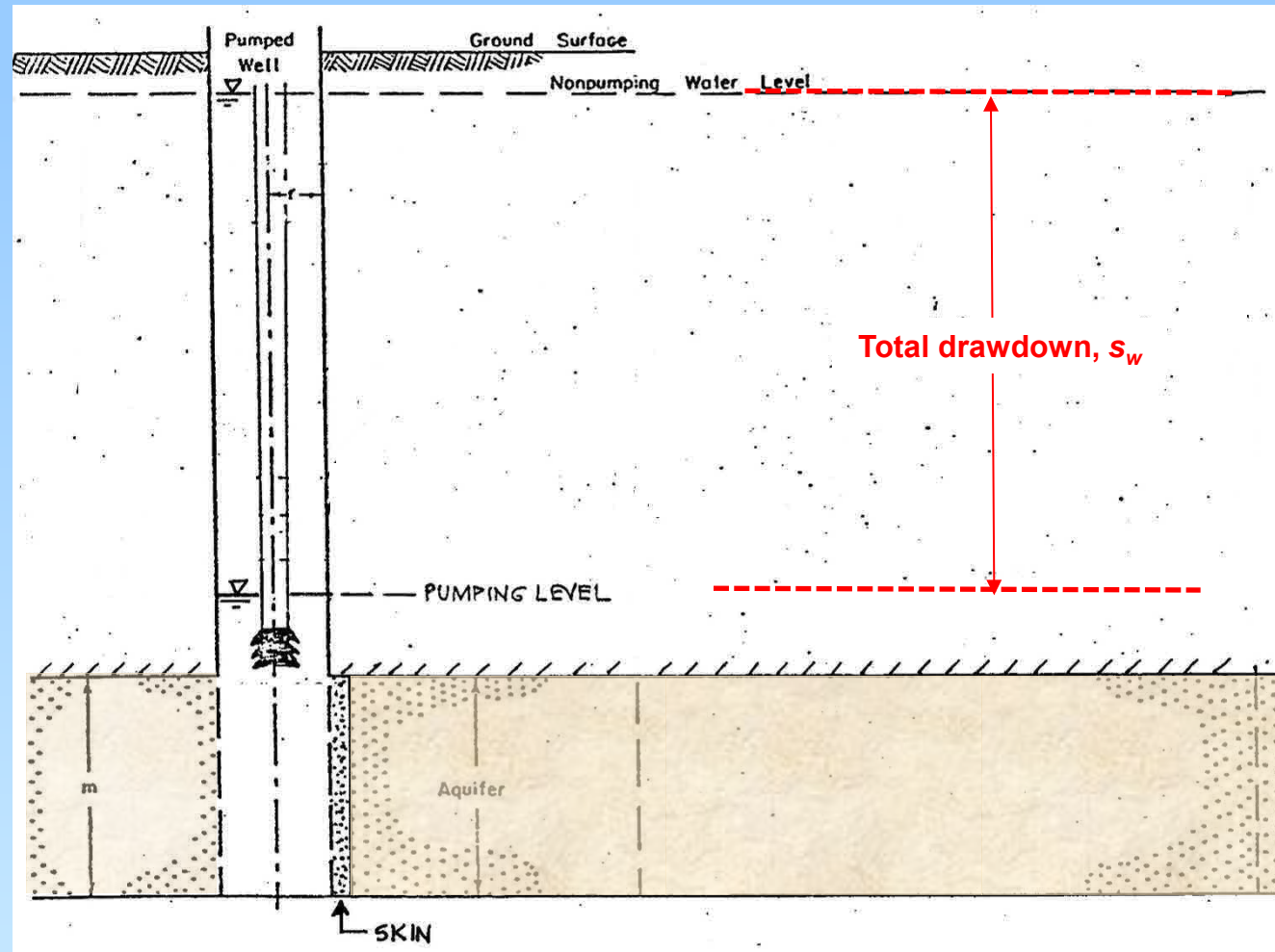
The Interpretation of Pumping Well Drawdowns

Christopher J. Neville
S.S. Papadopoulos & Associates, Inc.

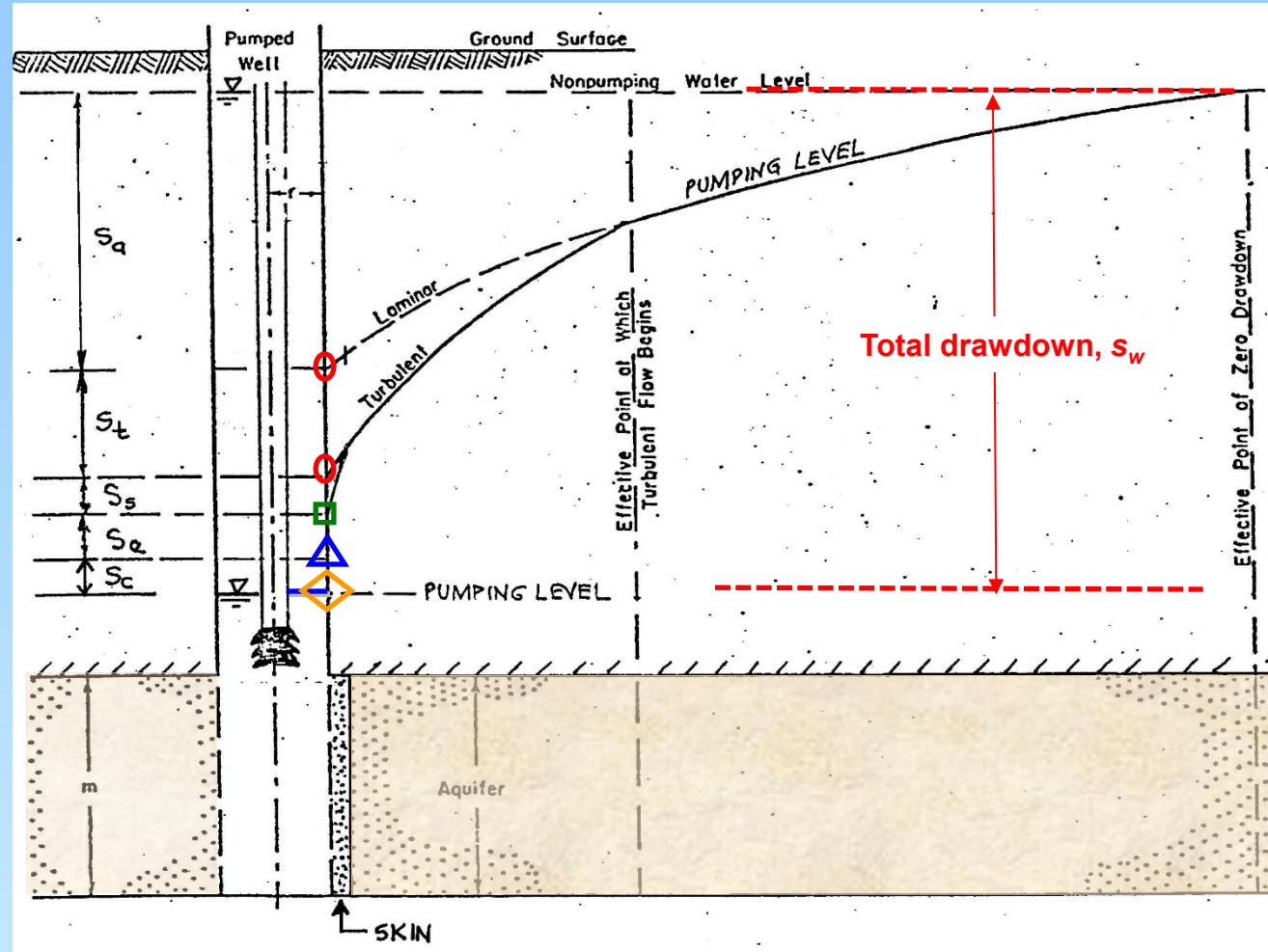
Outline

1. Overview of the hydraulics of pumping wells
2. Preliminary estimation of transmissivity
3. Extended interpretations of pumping well drawdowns
4. Step tests
5. Synthesis of pumping well and observation well drawdowns

Overview of the hydraulics of a pumping well



Components of the drawdown in a pumping well

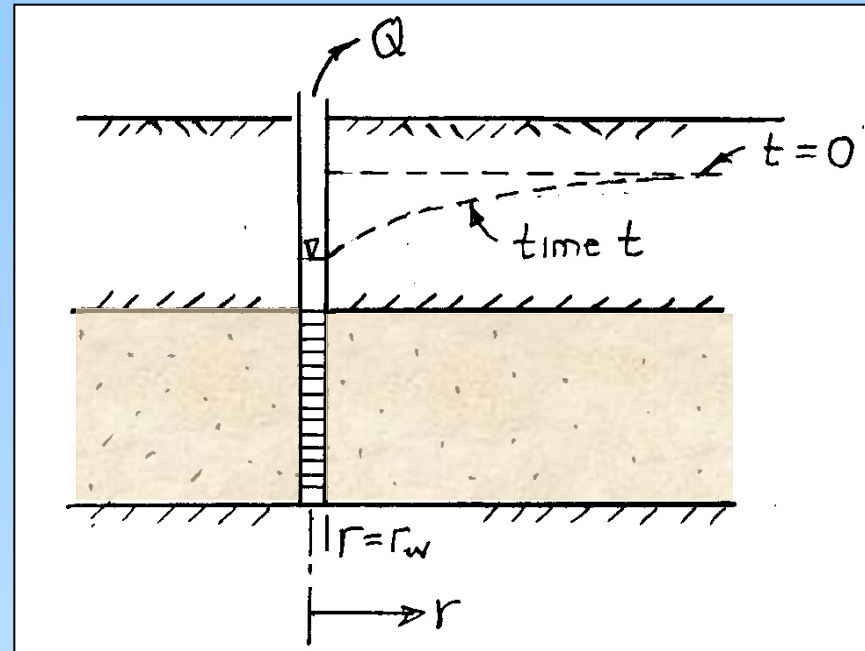


$$s_w(t) = s_a + s_t + s_s + s_e + s_c$$

Simplified model of total well drawdown

$$\begin{aligned}s_w(t) = & s_{formation} \\ & + \Delta s_{skin} \\ & + \Delta s_{turbulence}\end{aligned}$$

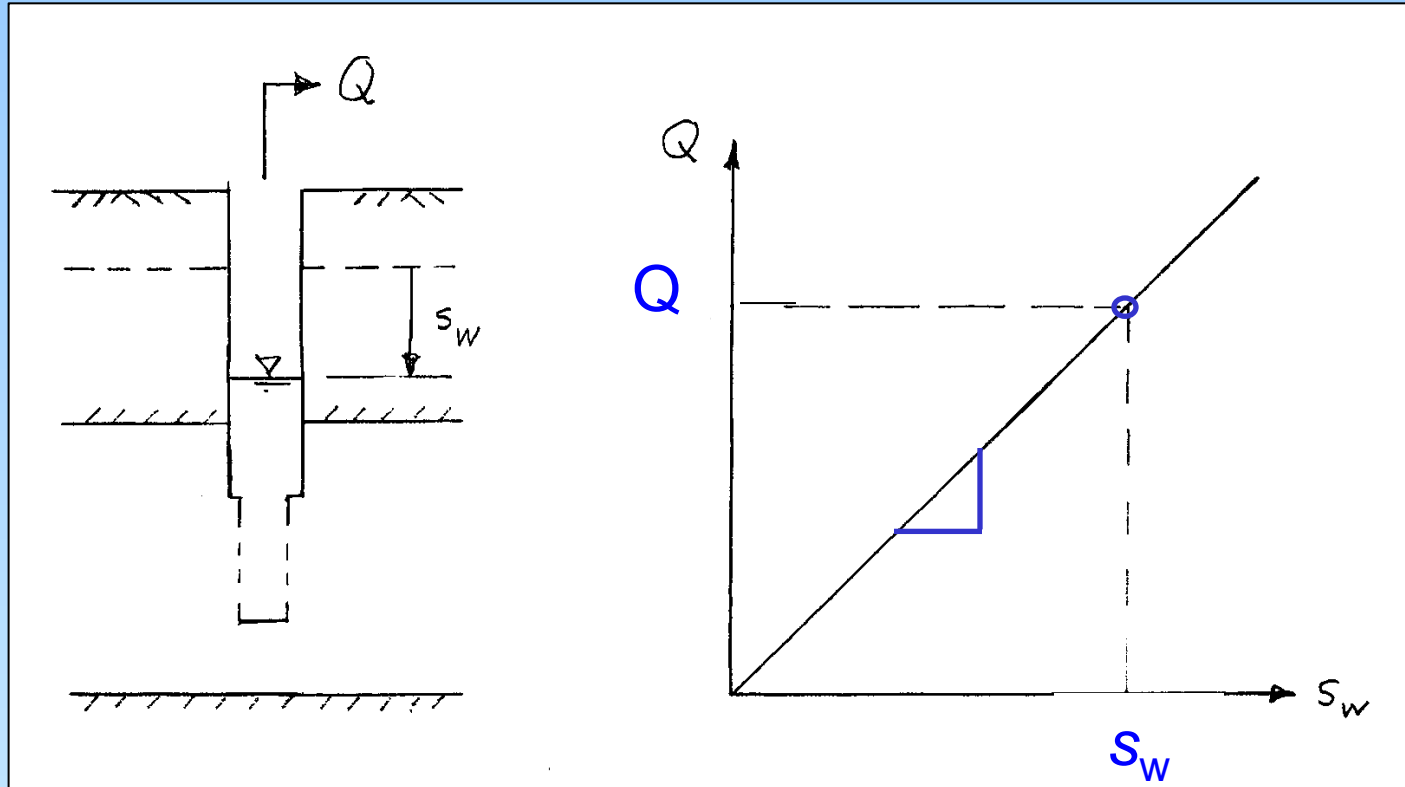
1. Drawdown in the formation, $s_{\text{formation}}$



$$s_{\text{formation}}(t) = Q \times F(r = r_w, t)$$

F is any aquifer conceptual model

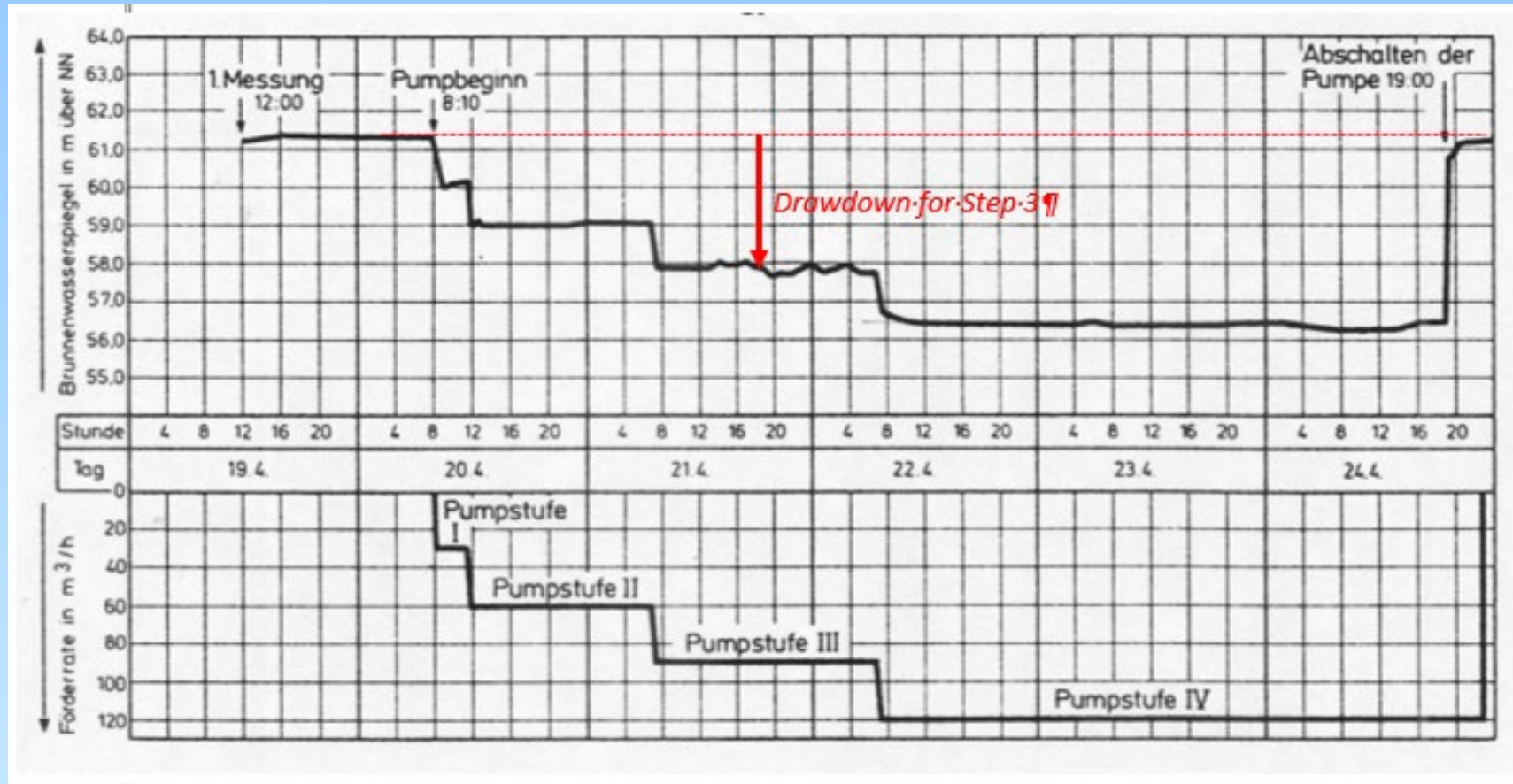
Drawdown in the formation: Ideal steady conditions

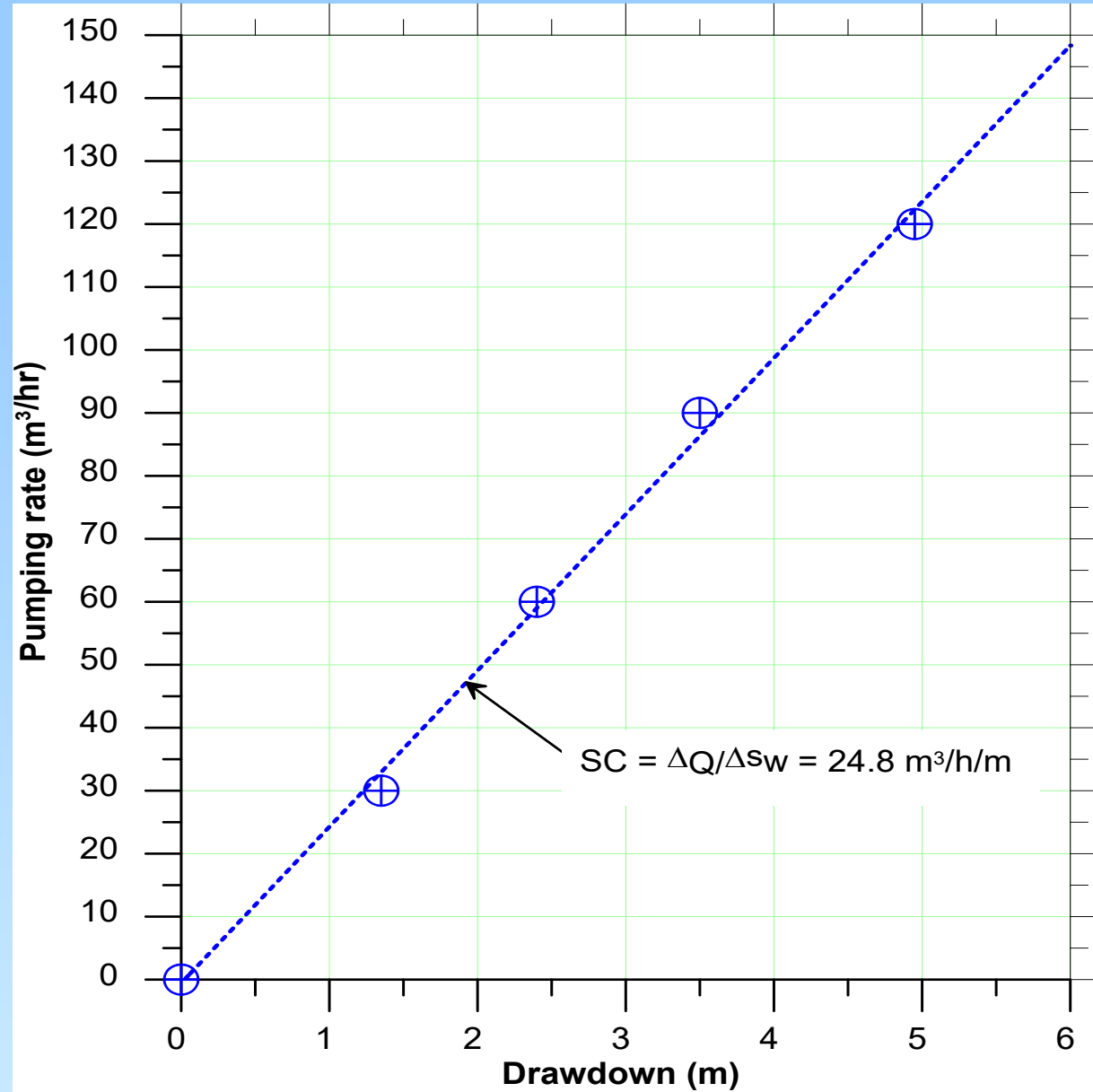


Specific capacity

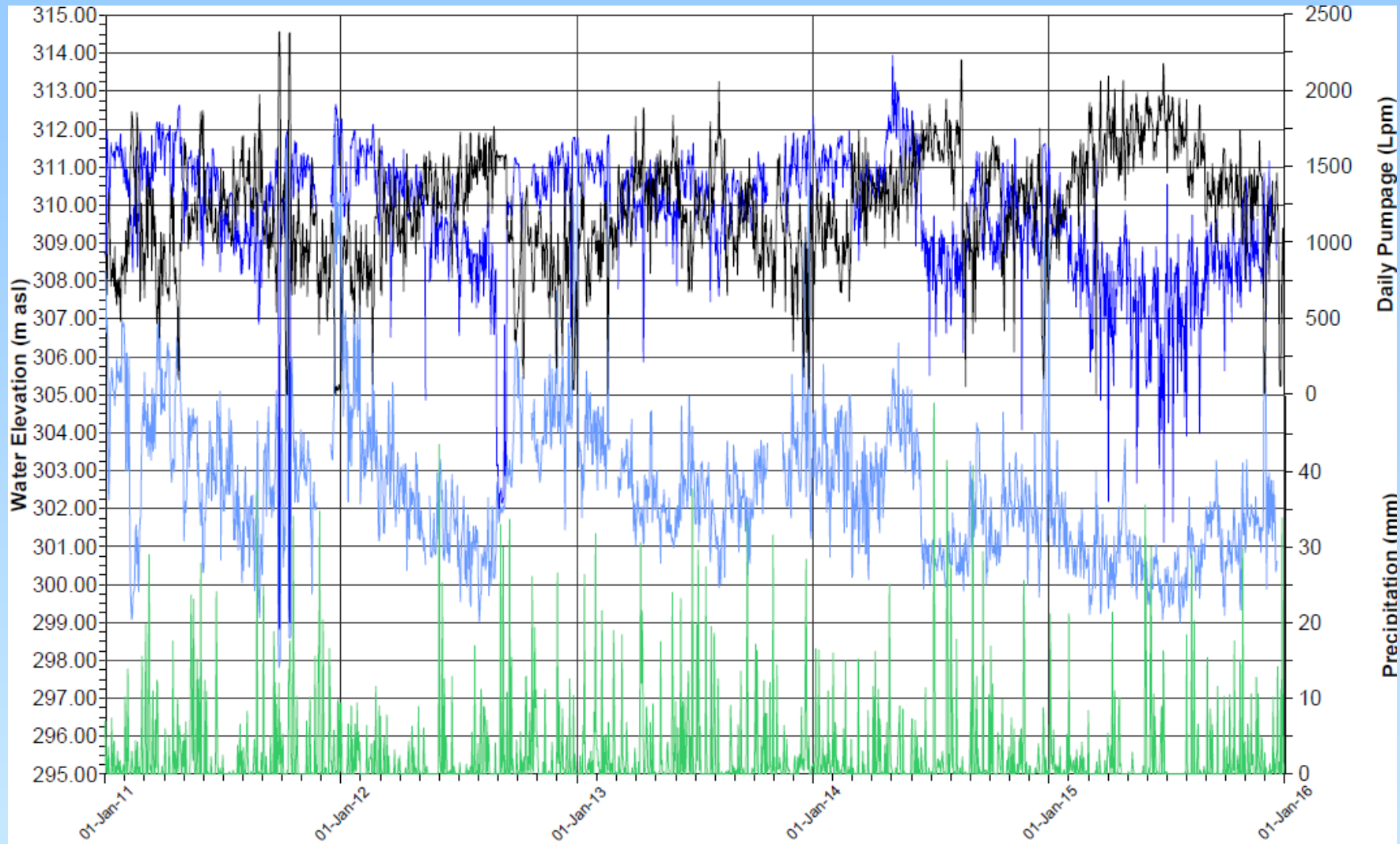
$$SC = \frac{Q}{s_w} = \frac{\Delta Q}{\Delta s_w}$$

Example: Linnich, Germany

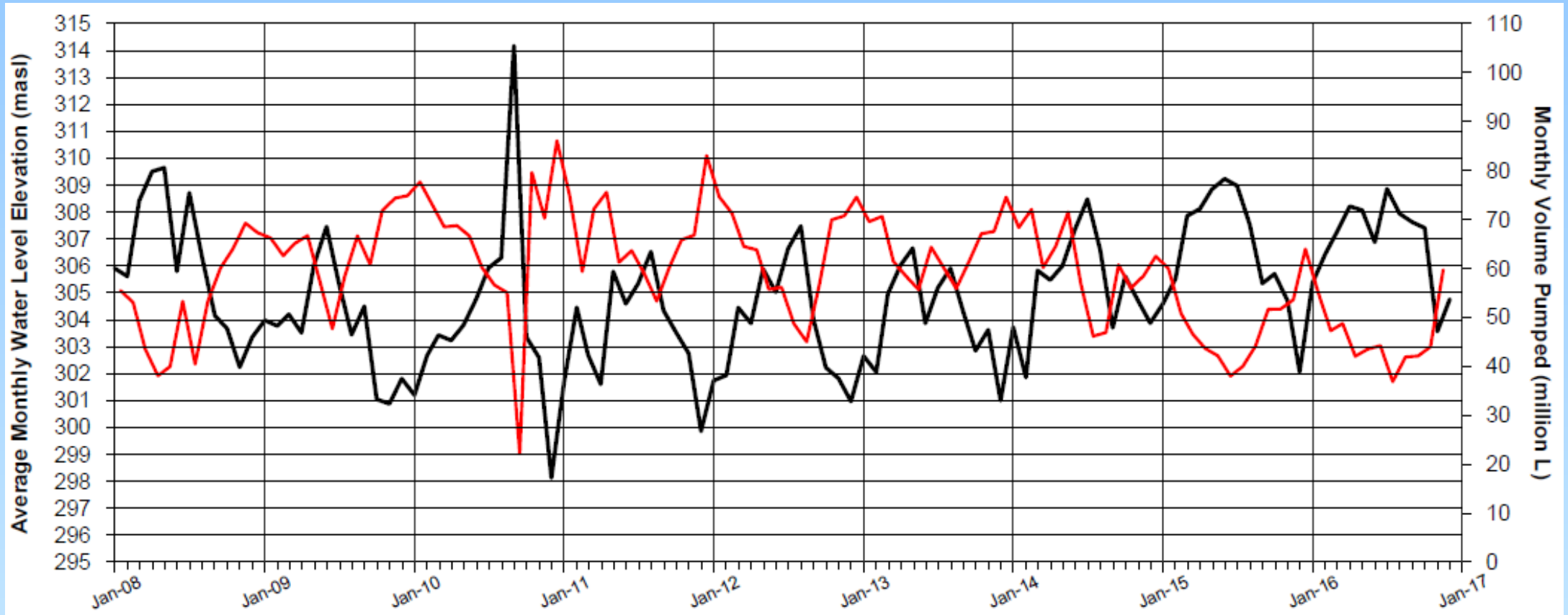


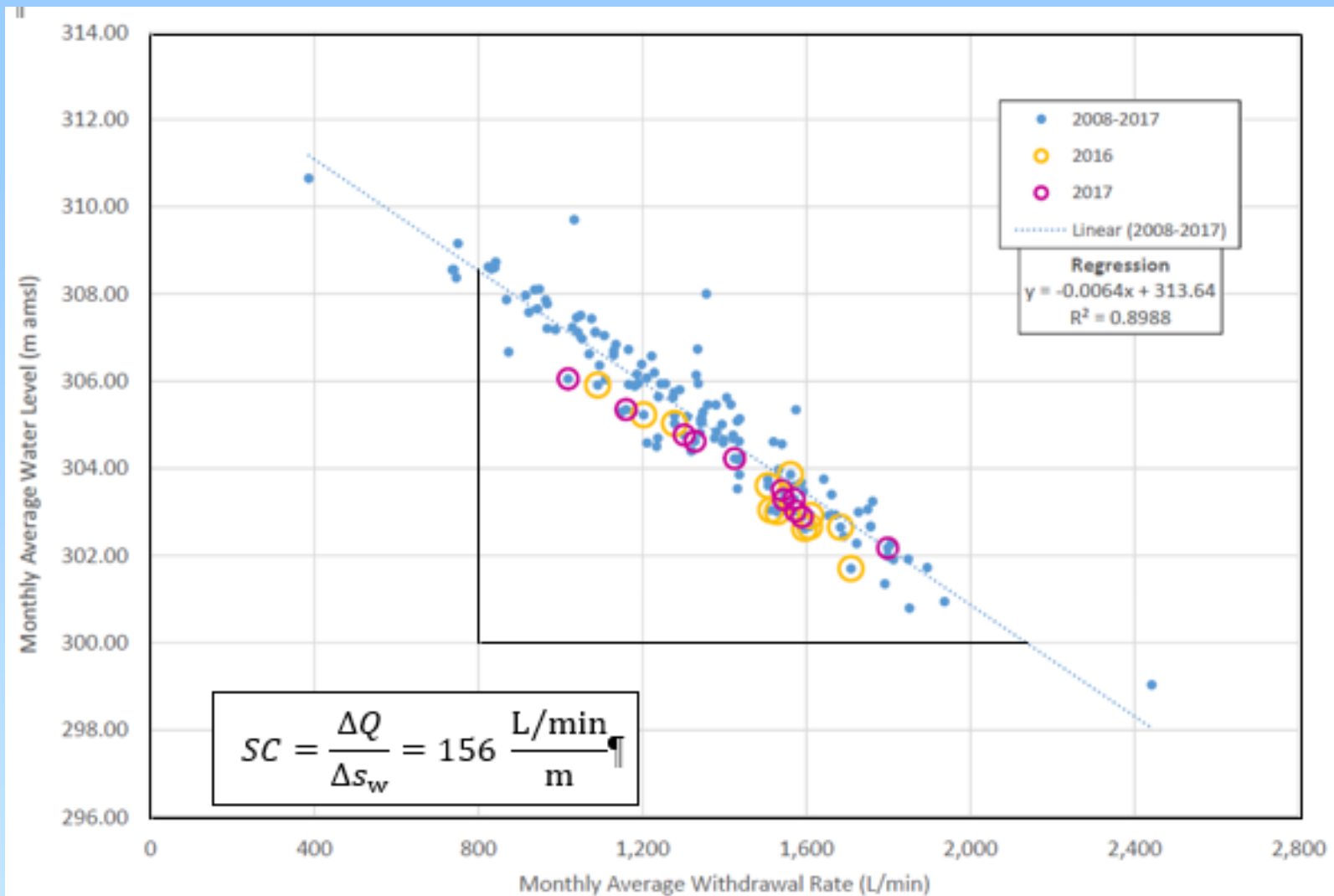


Example: TW3-80

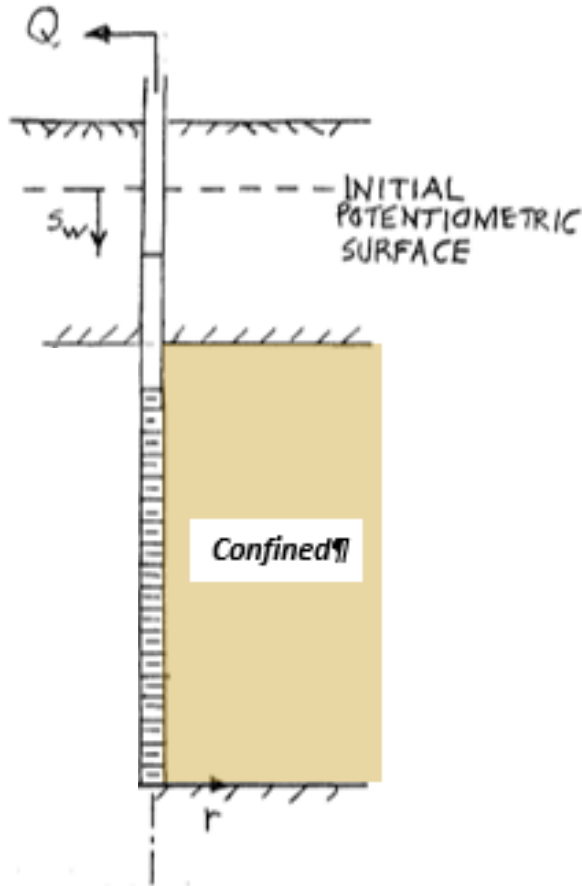


TW3-80 (2)





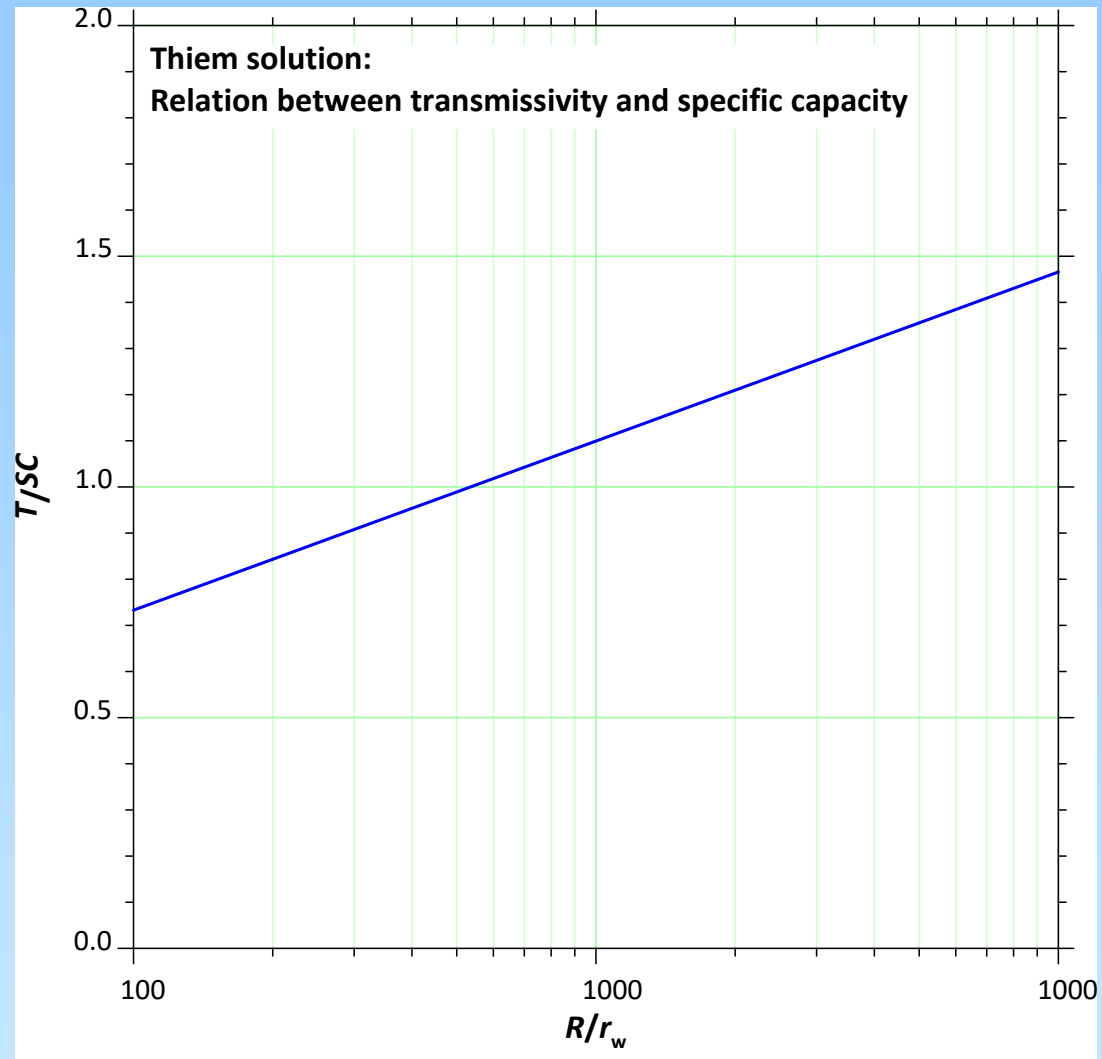
Estimation of transmissivity for ideal steady conditions



Thiem solution (steady conditions) for an “ideal” well:

$$Q = 2\pi T \frac{(h_R - h_w)}{\ln \left\{ \frac{R}{r_w} \right\}}$$

$$SC = \frac{Q}{s_w} = \frac{2\pi}{\ln \left\{ \frac{R}{r_w} \right\}} T$$

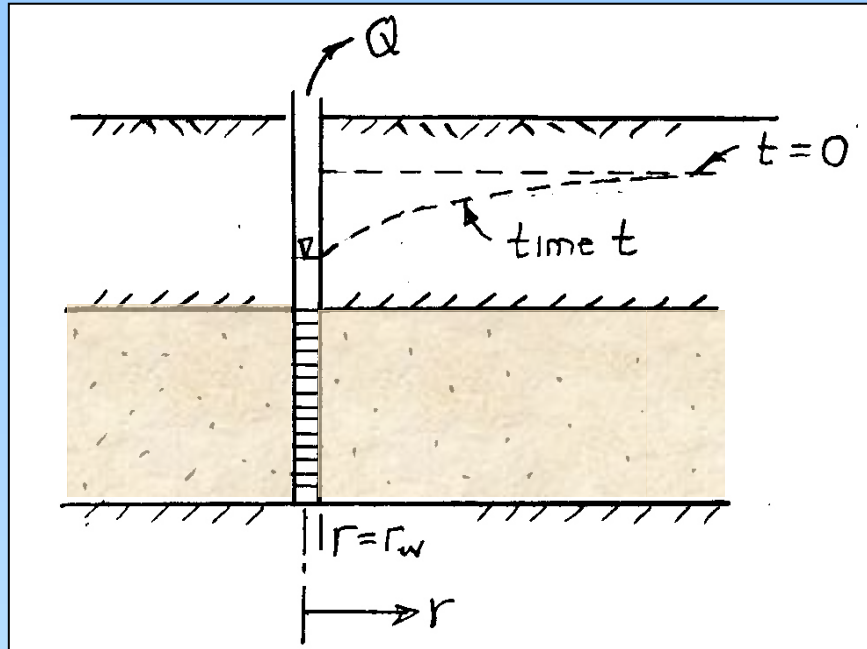


$$\frac{T}{SC} = \frac{\ln \left\{ \frac{R}{r_w} \right\}}{2\pi}$$

Given the specific capacity, SC:

$$T \approx SC$$

Drawdown in the formation: Ideal transient conditions



Theis (1935) evaluated at r_w

$$s_{\text{formation}}(t) = \frac{Q}{4\pi T} W \left\{ \frac{r_w^2 S}{4Tt} \right\}$$

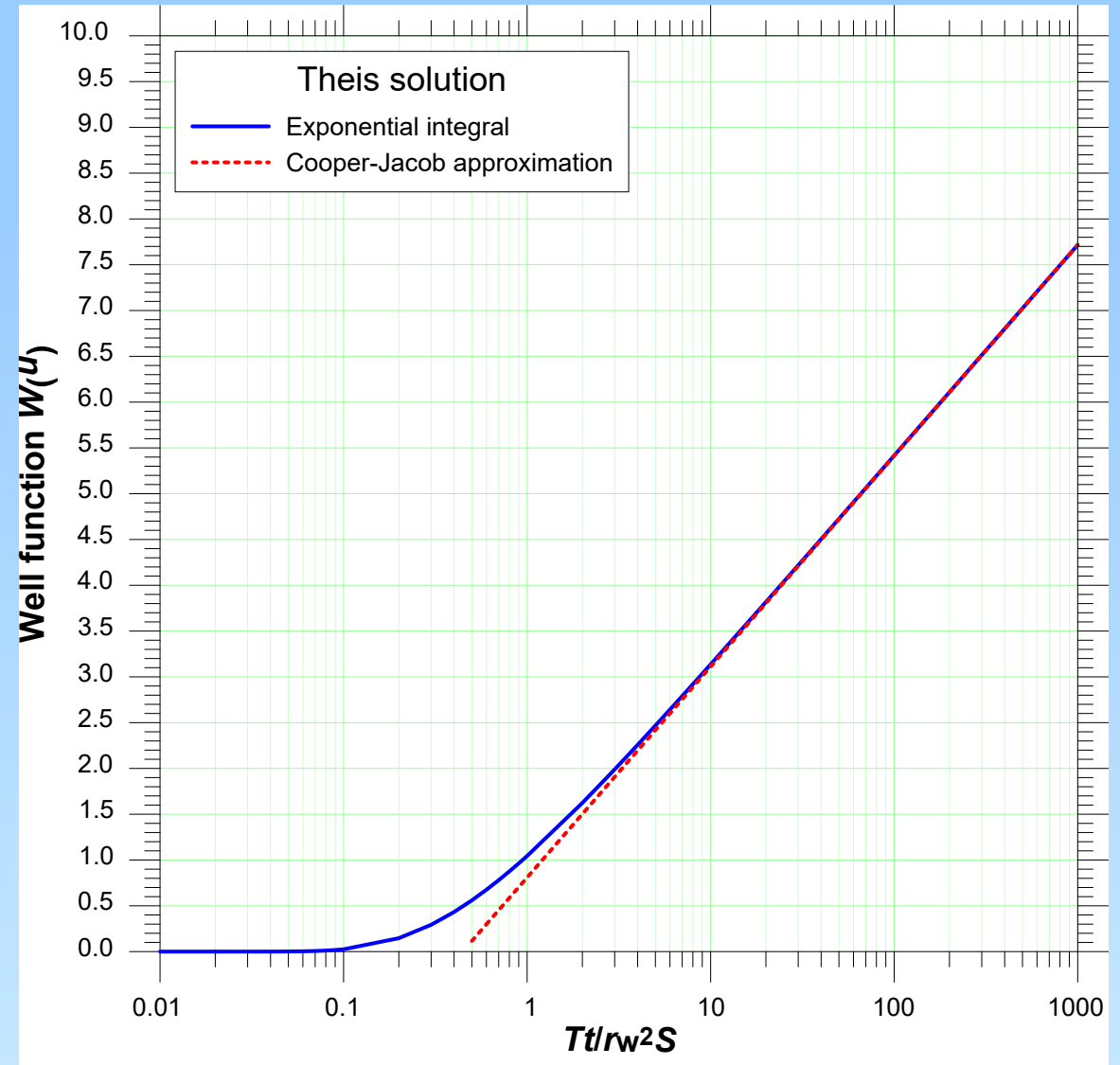
For all but the earliest times:

$$s_{\text{formation}}(r_w, t) \cong \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right]$$

The Cooper-Jacob approximation is reliable for values of $1/u > 10$.
For the pumping well we have:

$$\frac{1}{u_w} = \frac{4Tt}{r_w^2 S} > 10$$

$$t > 2.5 \frac{r_w^2 S}{T} \quad \text{Brief!}$$



**First-cut estimation of transmissivity
from the specific capacity, SC**

Let us assume that the drawdown is due entirely to head losses in the formation.

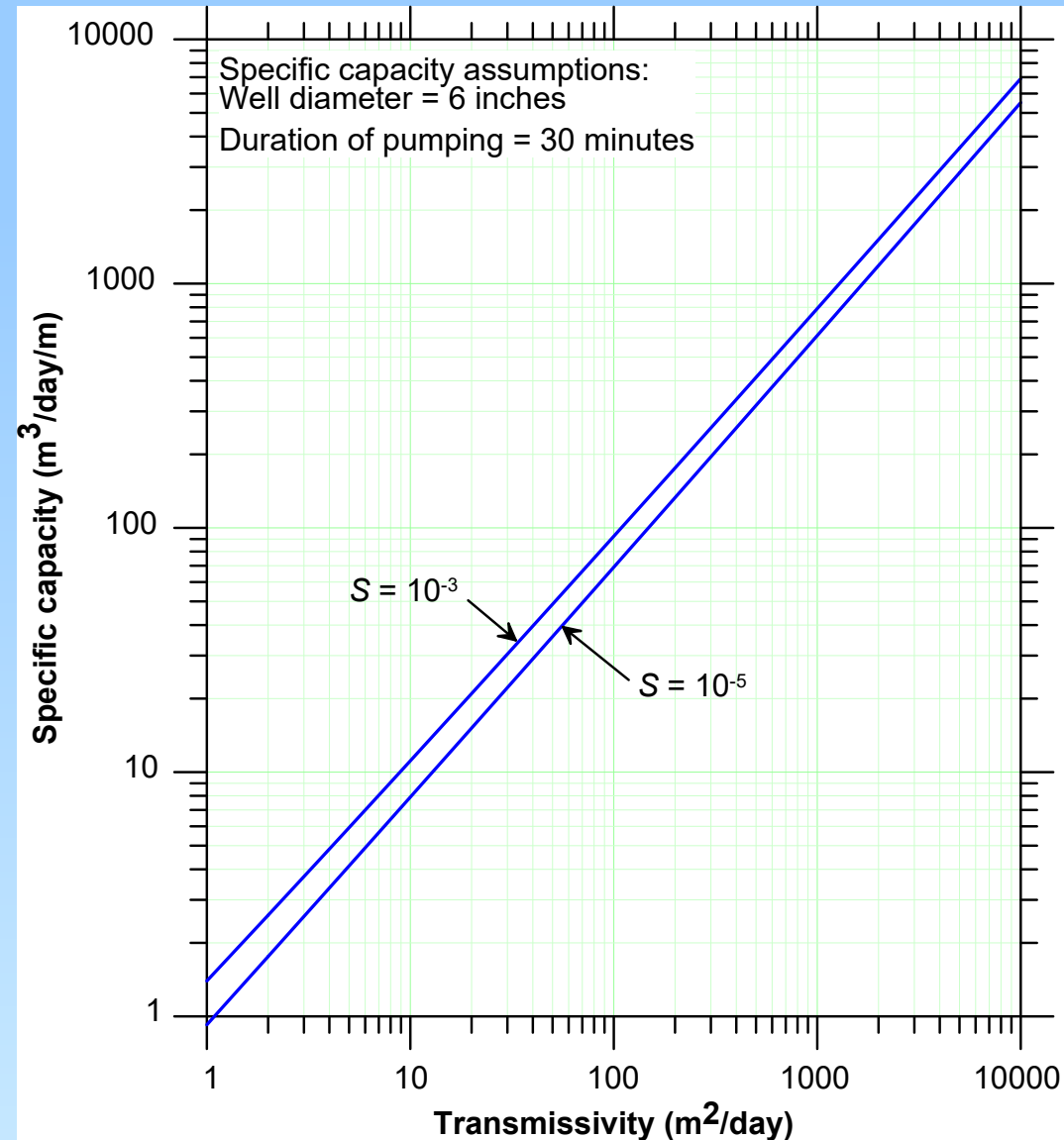
Theis solution:

$$s_{formation}(t) = \frac{Q}{4\pi T} W \left\{ \frac{r_w^2 S}{4Tt} \right\}$$

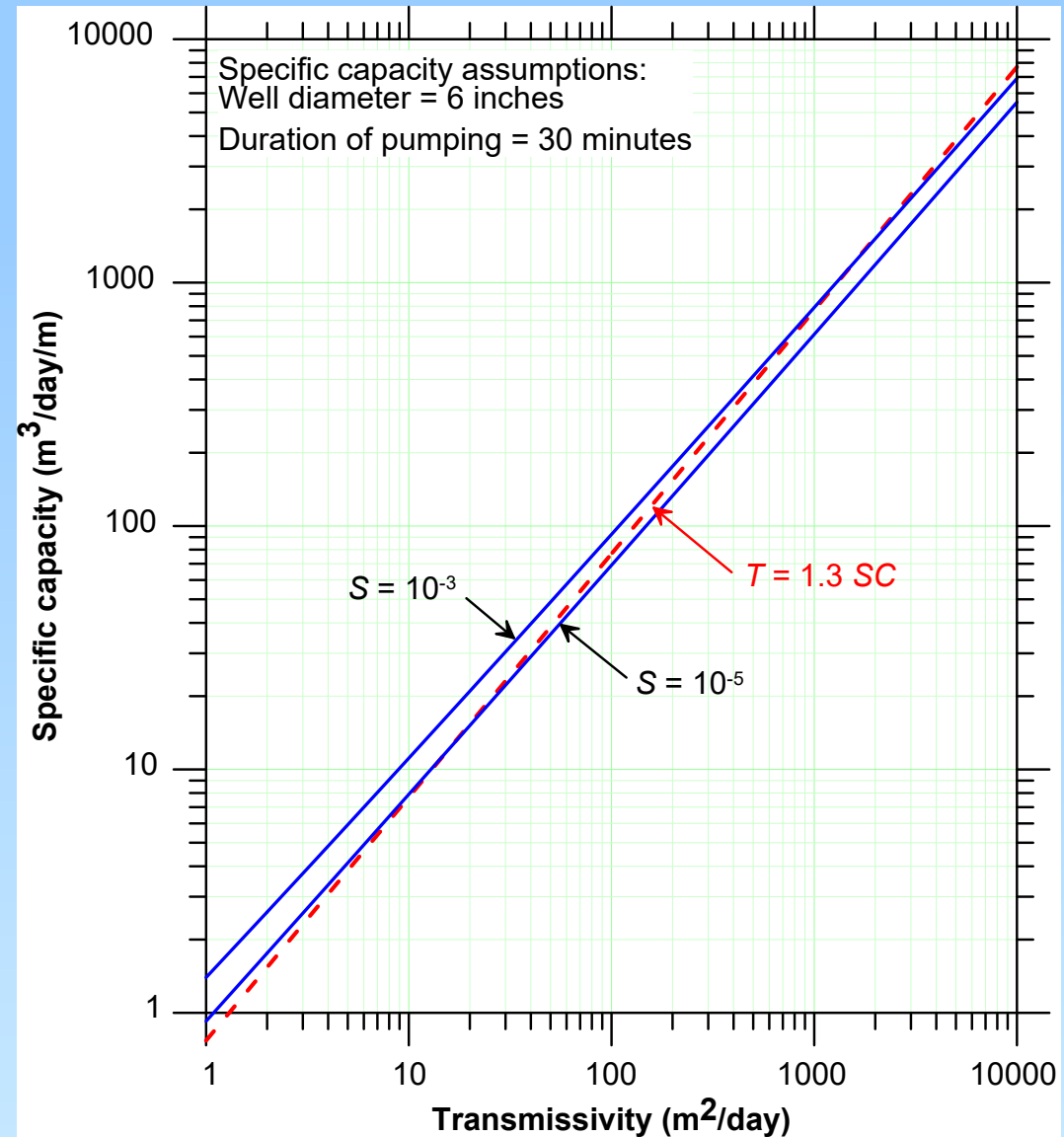
$$SC = \frac{Q}{s_{formation}(t)} = \frac{4\pi T}{W \left\{ \frac{r_w^2 S}{4Tt} \right\}}$$

If we know the specific capacity, the transmissivity T can be estimated by iteration. However, there is a simpler approach.

Specify r_w and t , calculate specific capacity for a range of transmissivities.



The leading coefficient will vary a bit,
depending on the duration of pumping.
But since this is a rough approximation,
 $T \sim SC$ is good enough.



WELL DIAGRAM			WELL MATERIAL																							
LOG 0' PCKD GRVL + BLDR 16' GRVL, SD+CY 23' BKN LMSN 50' 100' 150' 170' 20" CEMENT GROUT 23' 16" 39' BROWN LIMESTONE			Outer Casing: 20" dia., 0.375" Wall Thk. Matl.: STL Cemented from 0' to 39' Inner Casing: 16" dia., 0.375" Wall Thk. Matl.: STL Screen: Make: , dia., Opening & Matl.: Plug: Type: , Matl.: , Other: Gravel: Type: , Size: , Quantity:																							
WELL TEST DATA Preliminary Test Date: by Static Level: " below M.P. Pumping Rate IGM: Pumping Duration: hrs. min. Pumping Level at Test End: " Performance Plots: dd-t Dwg. dd-r Dwg. Step Test Final Test: Date MARCH 4/87 by N.B. Rated Well Capacity IGM Pumping Rate IGM 995 Static level 10' 10" Pumping level 28' 6" at 24 hrs. 0 min. Pump pressure: psi Main pressure: psi Shut off: AGH: psi; W.L.: " Clear Well Depth from B.P.: " Air Line: "			PUMP & MOTOR DATA Pump Make: Rating: IGM @ TH Head: Type: S.N. Column: X X "; Shaft Mtl.: Bowl: Stage: Curve: Suction: " dia. " Long Special: Zinc Sleeves: Taped Oil Line Other: Motor Make: Frame: SN: HP: ph, hz, rpm, Volts Bearing No. Upper: Lower: Special Equipment																							
WELL REVISIONS AND REHABILITATION <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>DATE</th> <th>WORK DONE</th> <th>BY</th> </tr> </thead> <tbody> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> <tr><td> </td><td> </td><td> </td></tr> </tbody> </table>			DATE	WORK DONE	BY																			International Water Supply Limited SASKATOON - BARRIE - MONTREAL CLIENT: REGION OF WATERLOO WELL NO: WILLARD WELL DRILLED BY: N. BARNHARDT DATE: FEB. 87 DRAWN: M.P. INSTALLED BY: DATE: DATE: APRIL 87		
DATE	WORK DONE	BY																								

- $Q = 995 \text{ l/gpm (6514 m}^3\text{/d)}$
- Static WL = 10' 10"
- Pumping WL = 28' 6" (@ 24 hrs)

$$s_w = 17.67 \text{ ft (5.39 m)}$$

$$T \sim 1.5 (Q/s_w) = 1800 \text{ m}^2\text{/d}$$

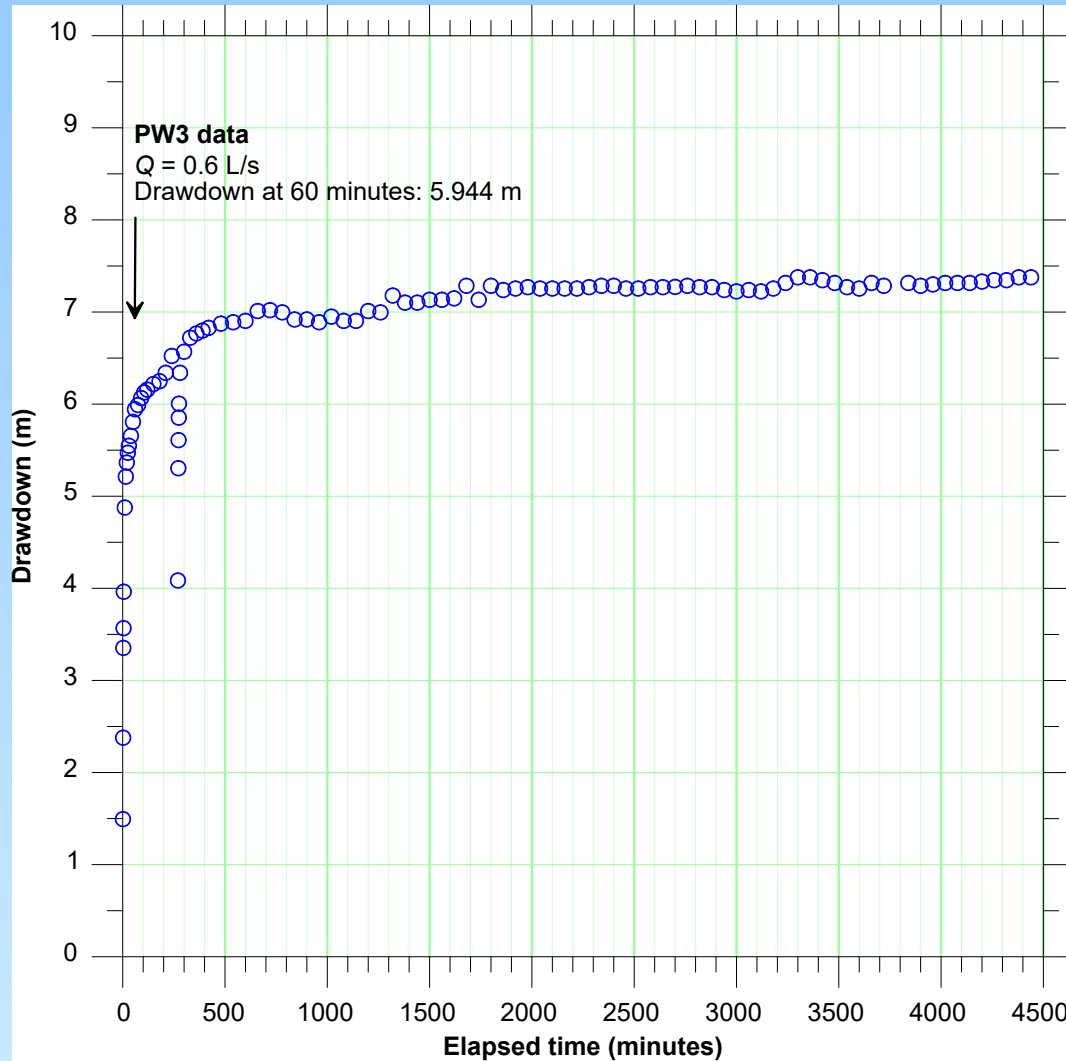
$$T \cong SC$$

... is handy

but ...

we have made the big assumption that the only source of the drawdown in the pumping well is head losses in the formation. For this reason, we refer to this transmissivity estimate as a “first-cut” or *reconnaissance-level* estimate.

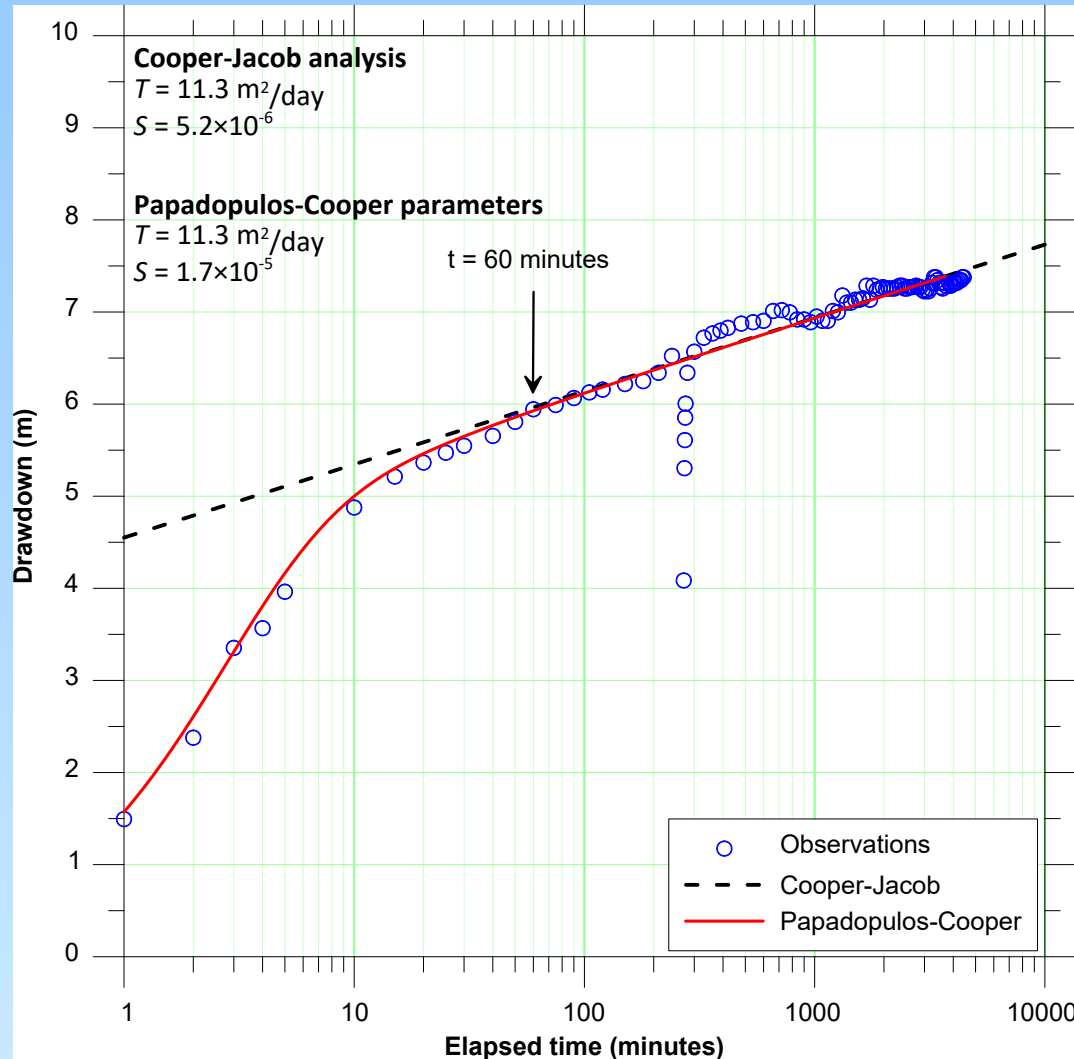
Example: Rosemont, Ontario



$$SC = \frac{(51.8 \text{ m}^3/\text{d})}{(5.94 \text{ m})} = 8.7 \frac{\text{m}^2}{\text{d}}$$

$$T \approx 1.3 SC = 11.3 \text{ m}^2/\text{d}$$

More rigorous analyses



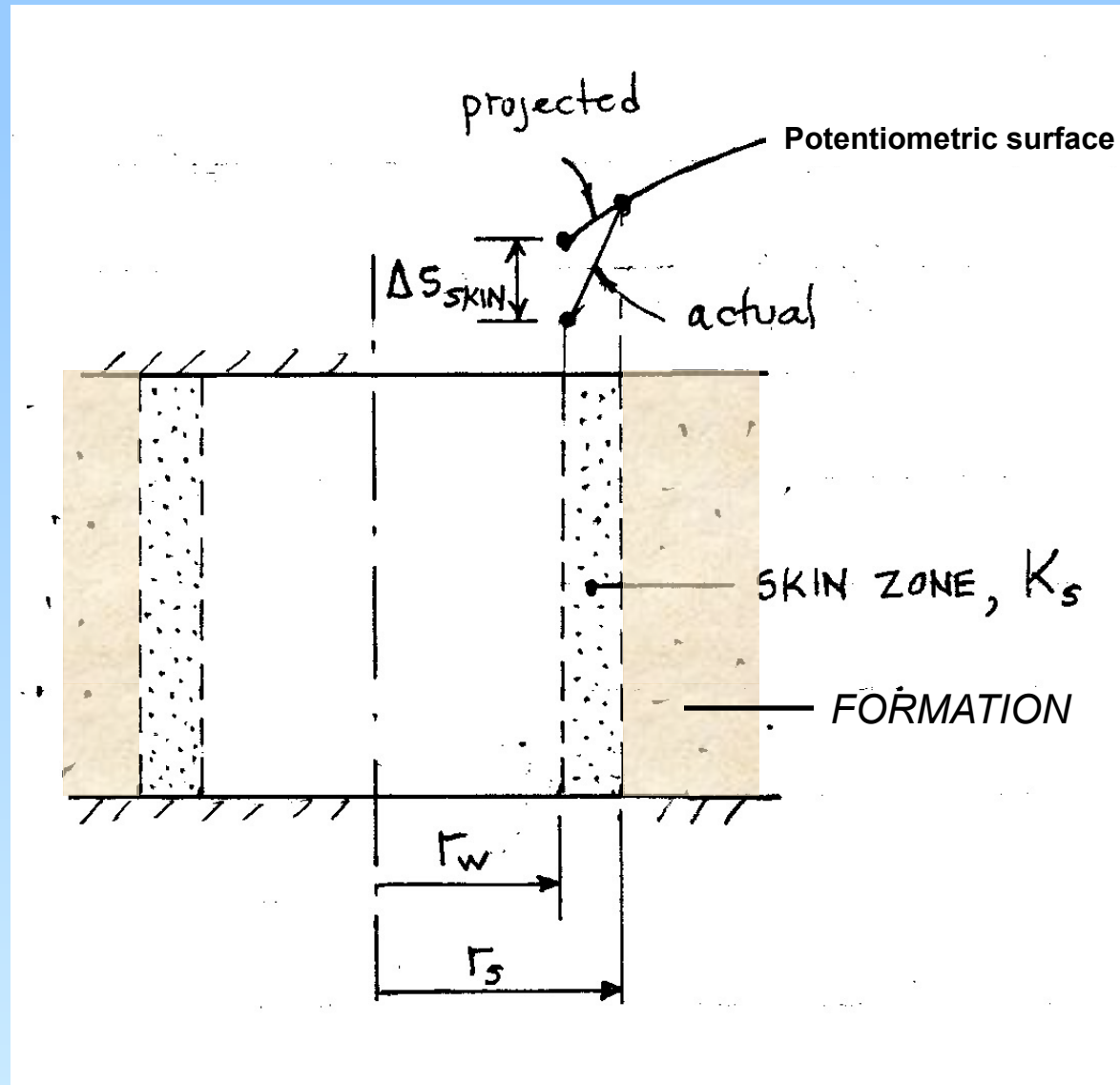
In this case, the close match between the approximate and more rigorous estimates of transmissivity are not by accident.

1. The “instantaneous” drawdown is relatively small.
2. The specific capacity is estimated from the drawdown after wellbore storage effects have dissipated.
3. The drawdowns over the duration of the test are matched closely with the Theis model.

Additional well losses

- Skin losses
- Additional losses due to partial penetration
- Nonlinear formation and/or in-well losses

Skin losses, Δs_{skin}



Characteristics of skin losses

1. Established relatively early;
2. Constant through time; and
3. Directly proportional to the pumping rate, Q .

Ramey formulation

$$\Delta S_{\text{skin}} = \frac{Q}{4\pi T} 2S_w$$

S_w is the dimensionless skin factor

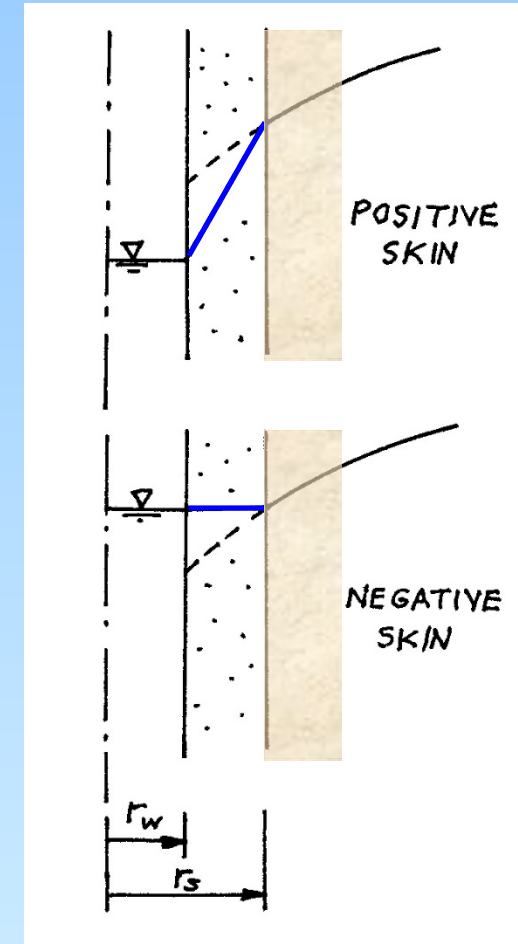
Interpretation of the dimensionless skin factor

$$S_w = \left(\frac{T_{\text{formation}} - T_{\text{skin}}}{T_{\text{skin}}} \right) \ln \left\{ \frac{r_s}{r_w} \right\}$$

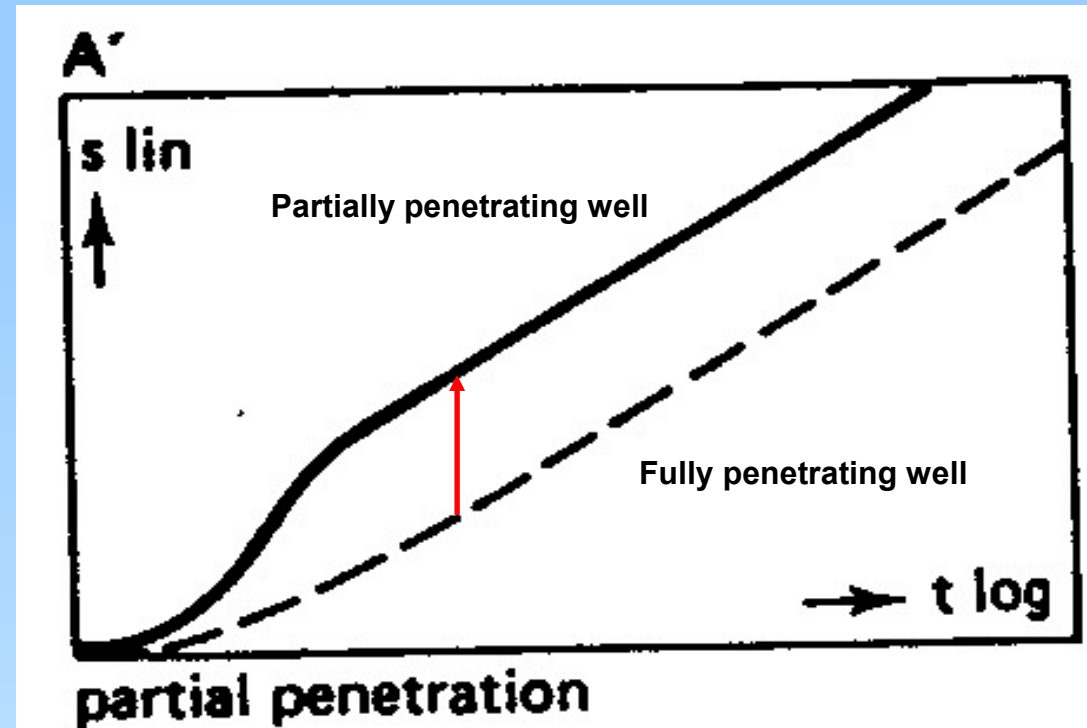
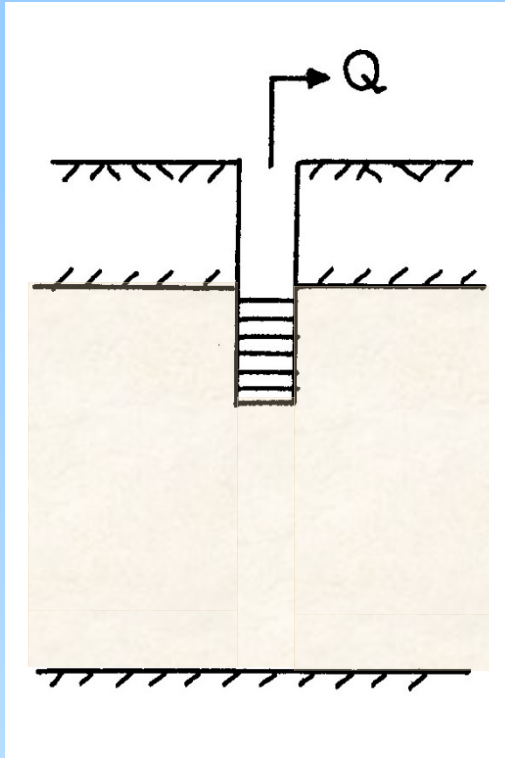
$$T_{\text{skin}} < T_{\text{formation}} \rightarrow S_w > 0$$

$$T_{\text{skin}} > T_{\text{formation}} \rightarrow S_w < 0$$

We can't know r_s , so S_w is treated as a lumped fitting parameter.



Additional head losses caused by partial penetration



- Established relatively early;
 - Constant through time; and
 - Directly proportional to Q .
- Same characteristics as skin losses

Additional head losses caused by partial penetration can be accounted for with a 'pseudo' skin factor, S_{PP} :

$$\Delta S_{PP} = \frac{Q}{4\pi T} 2S_{PP}$$

- Muskat (1932);
- Kozeny (1933); and
- Brons and Marting (1961)

Non-linear losses, $\Delta S_{turbulence}$

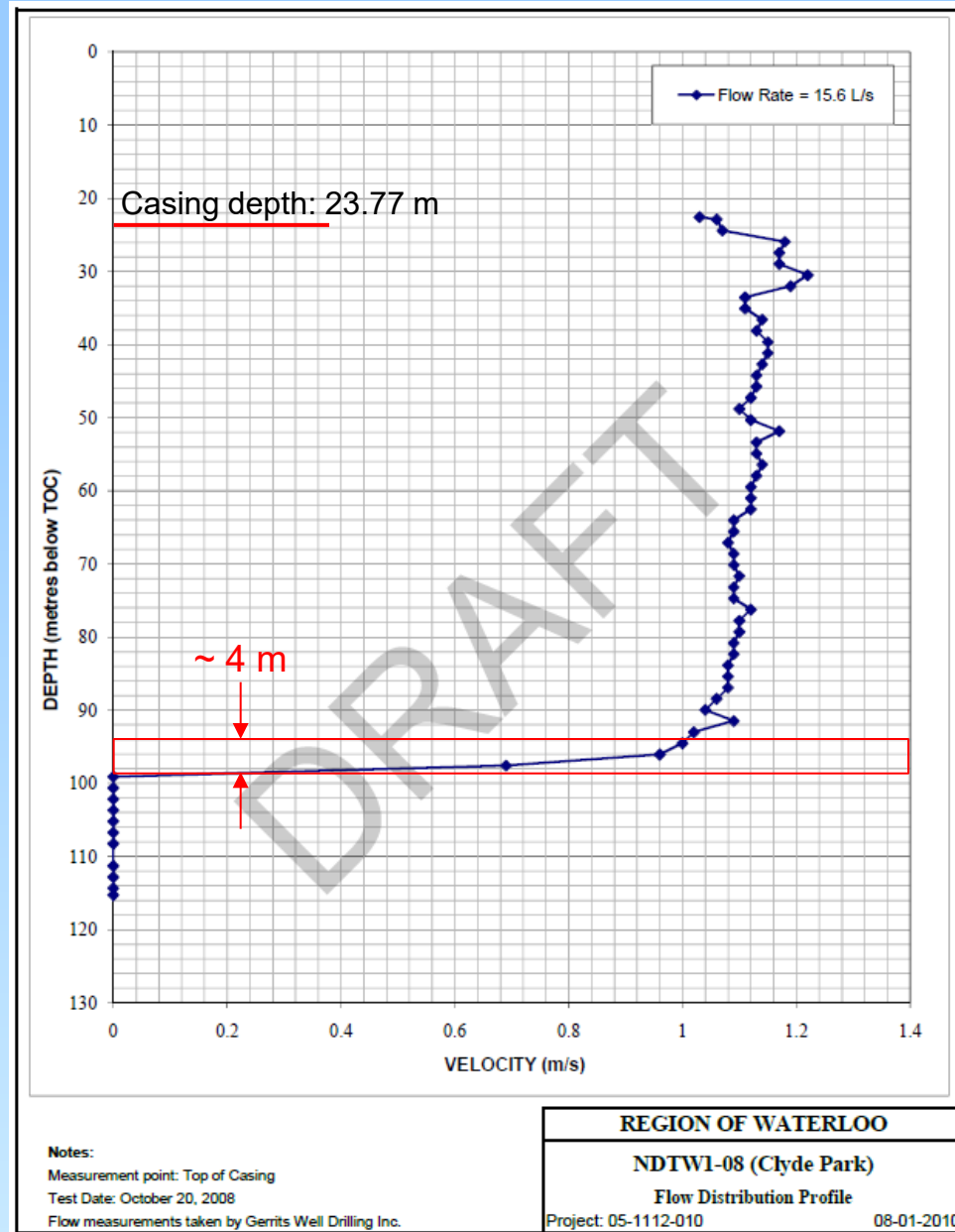
Jacob (1947) formulation:

$$\Delta S_{turbulence} = C Q^2$$

Rorabaugh (1953) generalization:

$$\Delta S_{turbulence} = C Q^P$$

Turbulence in the formation?



Estimating the nonlinear well loss coefficient, C

i. Rules-of-thumb: Walton (1970)

Condition of well	C (sec ² /ft ⁵)	C (day ² /ft ⁵)	C (sec ² /m ⁵)
Properly designed and developed	$C < 5$	$C < 6.7 \times 10^{-10}$	$C < 1900$
Mild deterioration	$C < 10$	$C < 1.3 \times 10^{-9}$	$C < 3800$
Well beyond rehabilitation	$C > 10$	$C > 1.3 \times 10^{-9}$	$C > 3800$

ii. Rigorous: Step tests

Watch units!

A slightly improved first-cut estimation of transmissivity from the specific capacity

Adjust drawdown to remove additional well losses:

$$s_{w-adj} = s_{w-obs} - \Delta s_{skin} - \Delta s_{turbulence}$$

$$SC_{adj} = \frac{Q}{s_{w-adj}} \Rightarrow T \approx 1.3 SC_{adj}$$

See Bradbury and Rothschild (1985) for the application of SC_{adj}

Analysis of transient drawdowns (1)

Constant-rate tests

The previous simplified analyses made use of only one drawdown at one time. We can develop more confidence in our analyses if we have a complete record of drawdowns in the pumping well.

General model to match the drawdowns:

$$s_w(t) = s_{formation}(r_w, t) + \Delta s_{skin} + \Delta s_{turbulence}$$

**“Extended Theis” model
(AQTESOLV Theis step test model)**

$$s_w(t) = \frac{Q}{4\pi T} W \left\{ \frac{r_w^2 S}{4Tt} \right\} + \frac{Q}{4\pi T} 2S_w + CQ^P$$

“Extended Theis” model (2)

$$\begin{aligned}s_w(t) &= \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right] + \frac{Q}{4\pi T} 2S_w + CQ^P \\&= \frac{Q}{4\pi T} \left[-0.5772 - \left(\ln \left\{ \frac{1}{t} \right\} + \ln \left\{ \frac{r_w^2 S}{4T} \right\} \right) \right] + \frac{Q}{4\pi T} 2S_w + CQ^P \\&= \boxed{2.303 \frac{Q}{4\pi T} \left[\log_{10} \left\{ 2.246 \frac{T}{r_w^2 S} \right\} \right] + \frac{Q}{4\pi T} 2S_w + CQ^P} \\&\quad + 2.303 \frac{Q}{4\pi T} \log_{10}\{t\}\end{aligned}$$

Example time-drawdown record

Specified parameters:

$$r_w = 0.05 \text{ m}$$

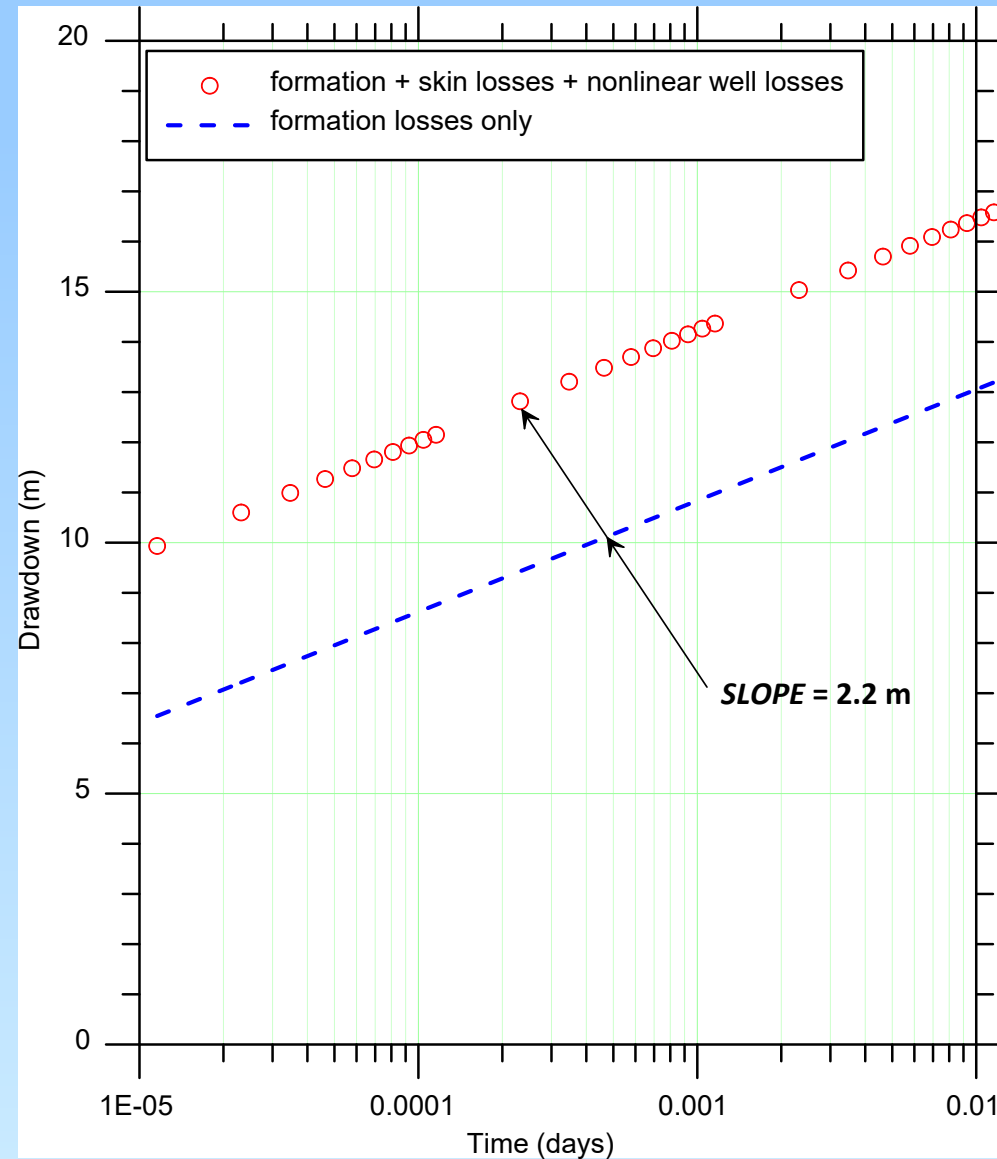
$$T = 8.6 \text{ m}^2/\text{d}$$

$$S = 1.0 \times 10^{-4}$$

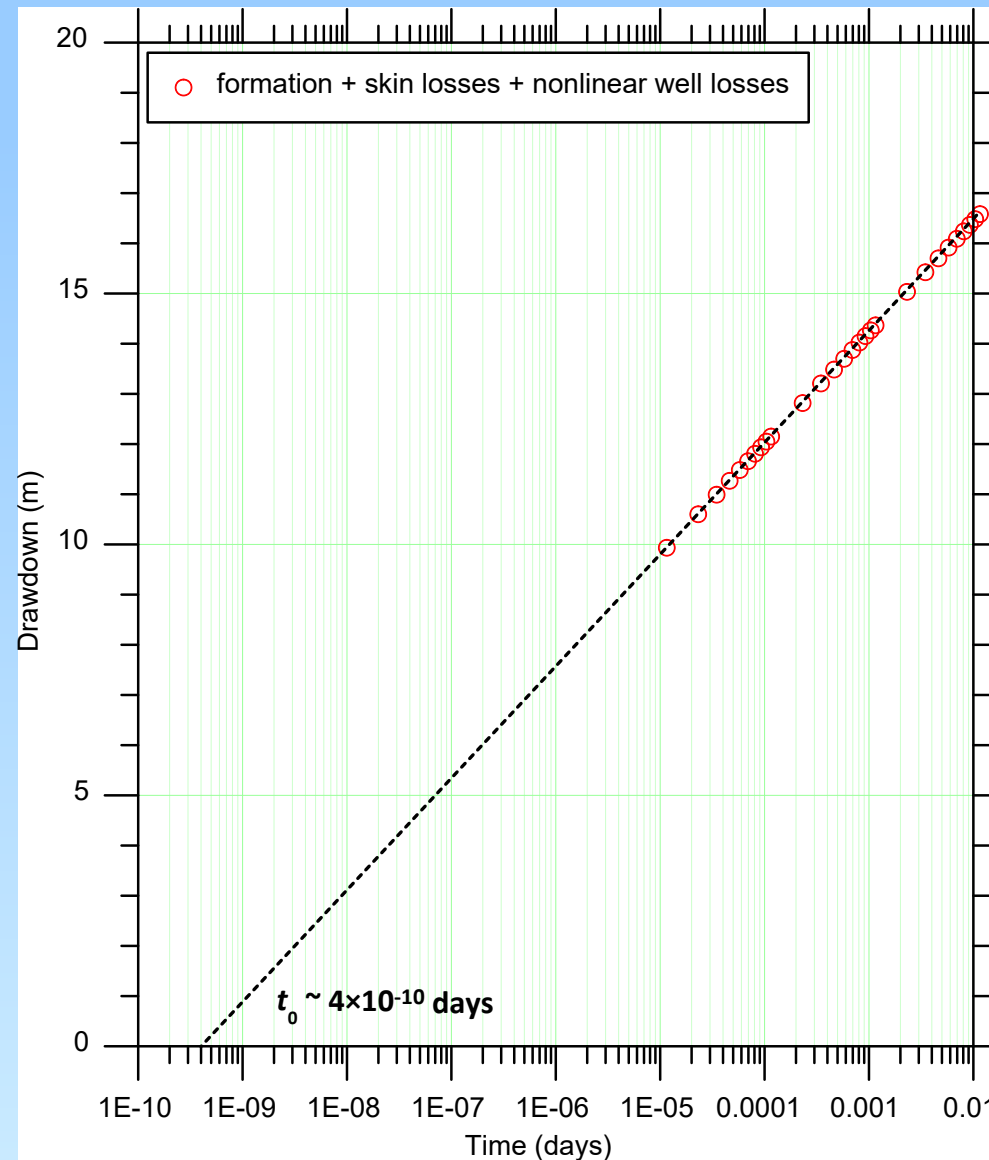
$$S_w = 0.52$$

$$C = 1.34 \times 10^{-4} \text{ m}^{-5}\text{d}^2$$

$$Q = 104.5 \text{ m}^3/\text{d}$$

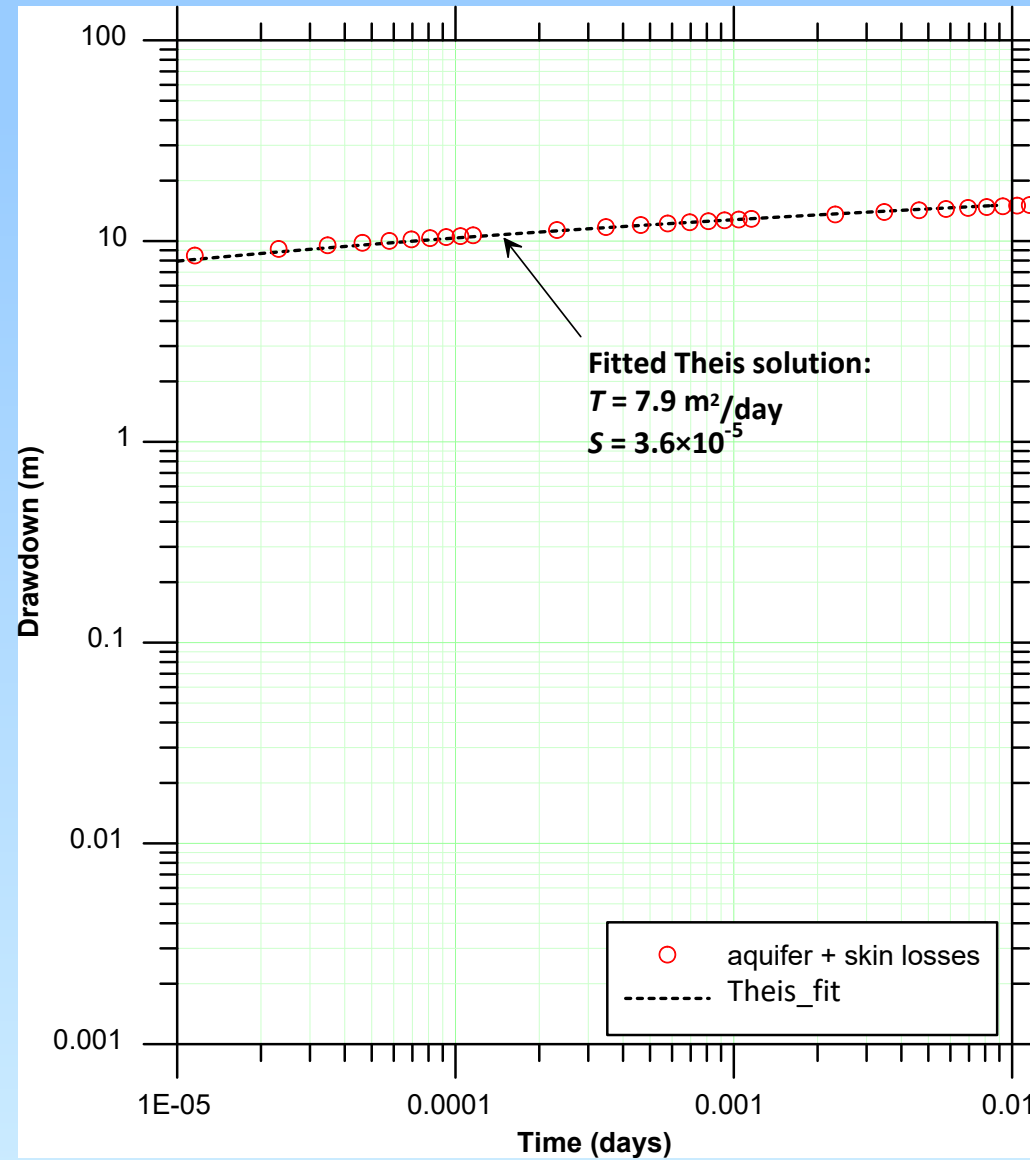


Diagnosis of additional well losses



$S_{\text{true}} = 1.0 \times 10^{-4}$
 $S_{\text{fit}} = 7.7 \times 10^{-7}$

Diagnosis of additional well losses: Log-log plot?



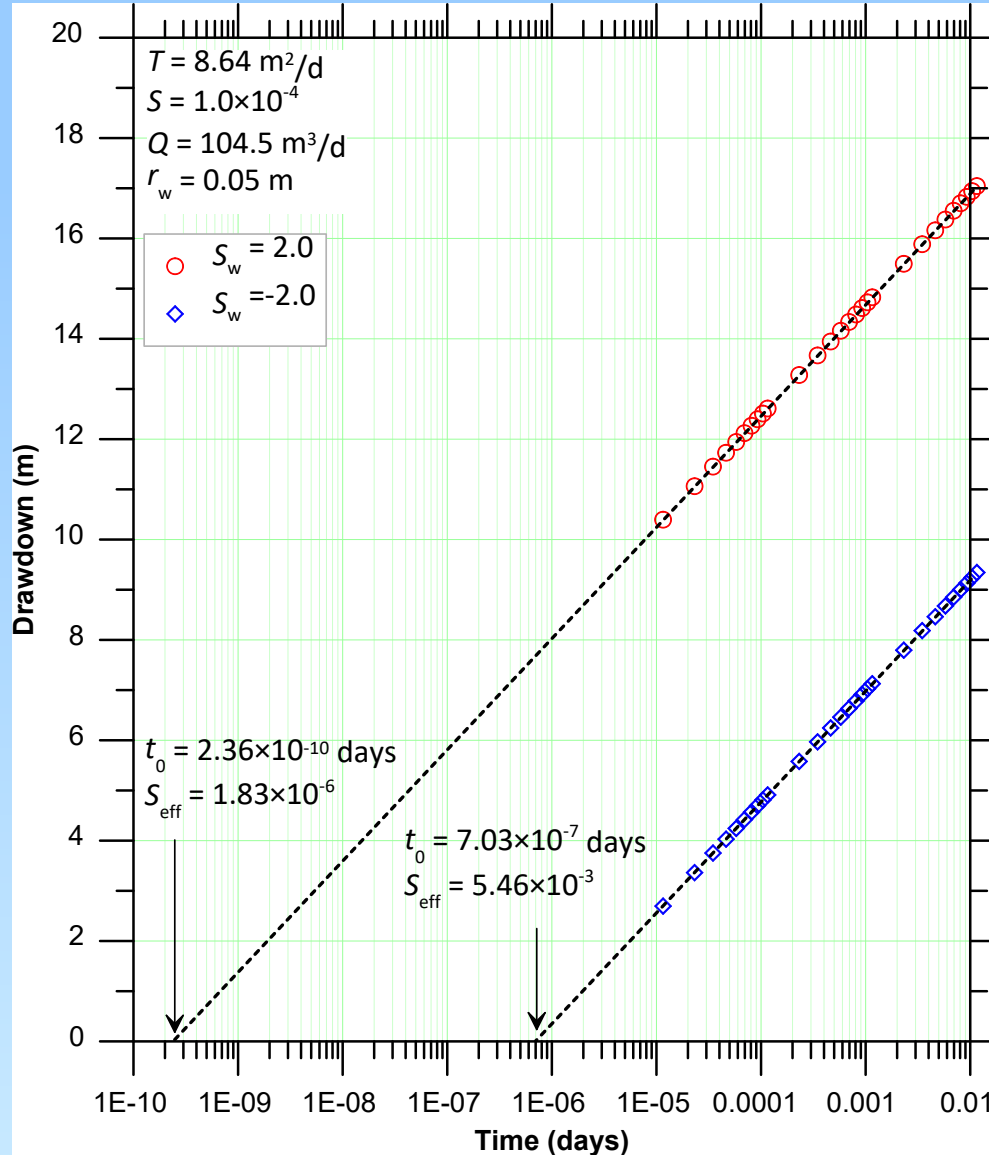
Recall:
 $T = 8.6 \text{ m}^2/\text{d}$
 $S = 1.0 \times 10^{-4}$

Further thoughts on the apparent storage coefficient

$$\begin{aligned}s_w(t) &= \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right] + \frac{Q}{4\pi T} 2S_w \\ &= \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right] + \frac{Q}{4\pi T} \ln \{ \text{EXP} \{ 2S_w \} \}\end{aligned}$$

$$s_w(t) = \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 (\text{SEXP} \{ -2S_w \})}{4Tt} \right\} \right]$$

Diagnosis of skin effects from the semilog plot (2)



$$S_{\text{eff}} = 2.246 \frac{T t_0}{r_w^2}$$

$$= \text{SEXP}\{-2S_w\}$$

Checks:

$$S_w = +2.0$$

$$S_{\text{eff}} = 1.8 \times 10^{-6}$$

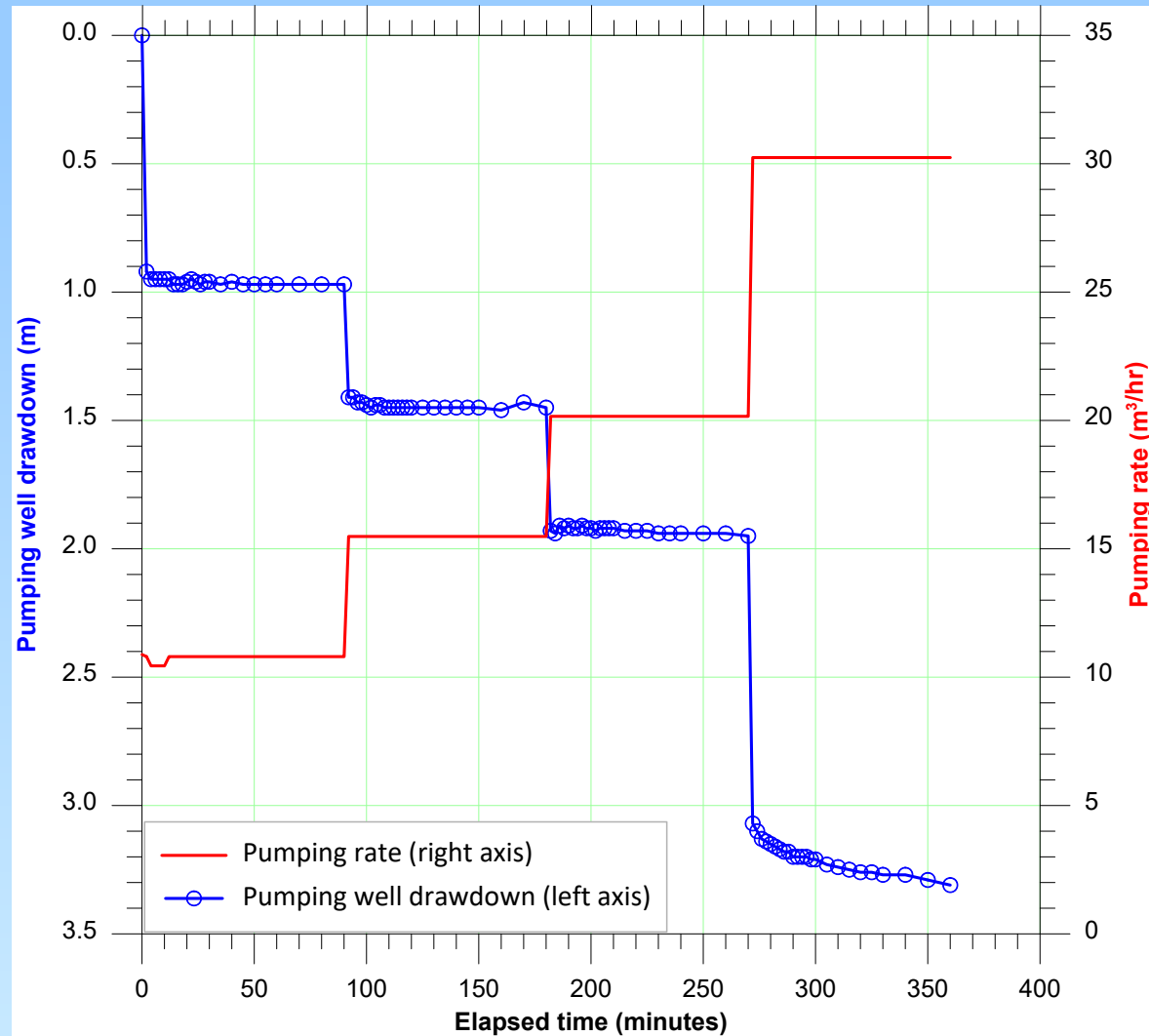
$$S_w = -2.0$$

$$S_{\text{eff}} = 5.5 \times 10^{-3}$$

Comments on the interpretation of pumping well drawdowns

1. A “strange” estimate of the storage coefficient derived from a Cooper-Jacob analysis points to something missing in the analysis.
2. Transmissivity estimates derived from Cooper-Jacob analyses are not affected by additional losses, but the results of Theis analyses are.
3. If we want to refine our interpretation of the additional losses, we require data from more than one pumping rate.

Step tests



Why conduct a step test?

1. Step tests are an essential part of the design of constant-rate pumping tests.
2. Step test are important for understanding the performance of production wells.
3. We can use the results of step tests to “correct” the pumping well drawdowns so that they can be analyzed in conjunction with the data from observation wells.

Diagnosis of nonlinear well losses: Hantush-Bierschenk plot

Assume stabilized conditions, with turbulent head losses given by the Jacob (1947) formulation:

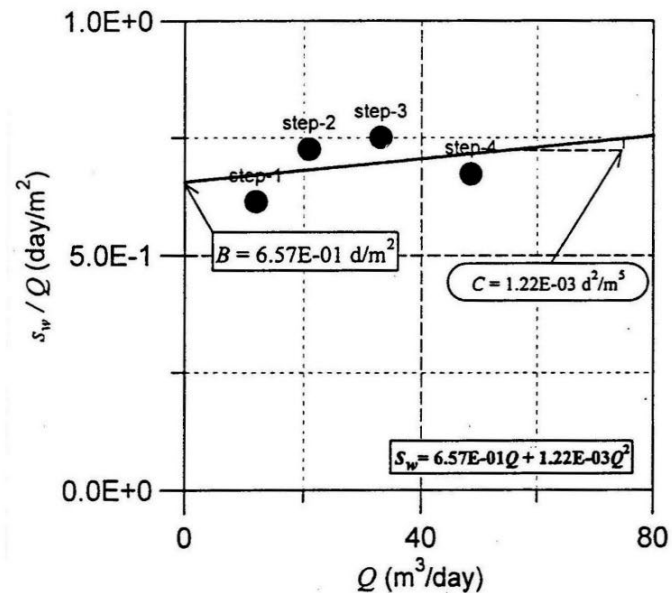
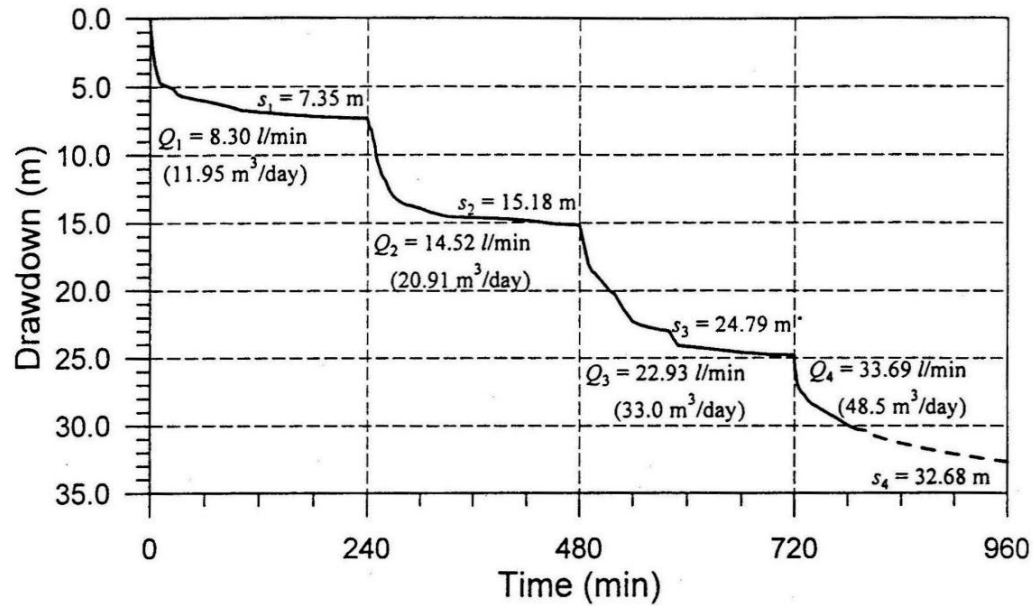
$$s_w = BQ + CQ^2$$

Divide through by Q :

$$\frac{s_w}{Q} = B + CQ$$

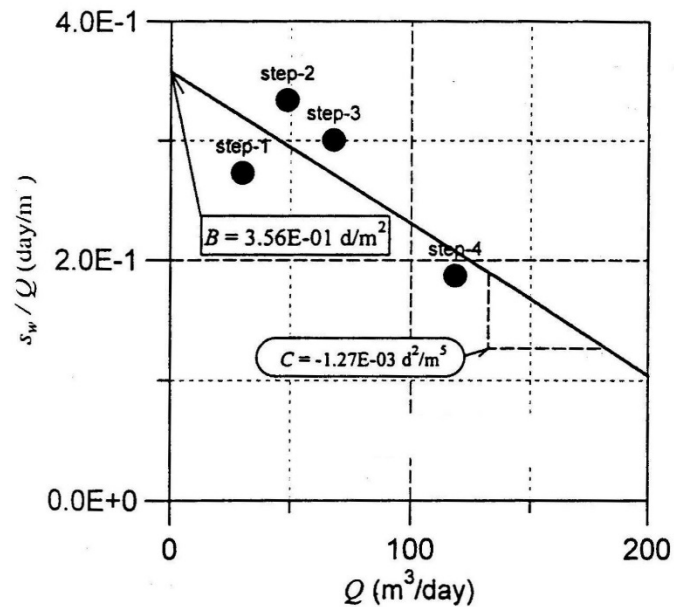
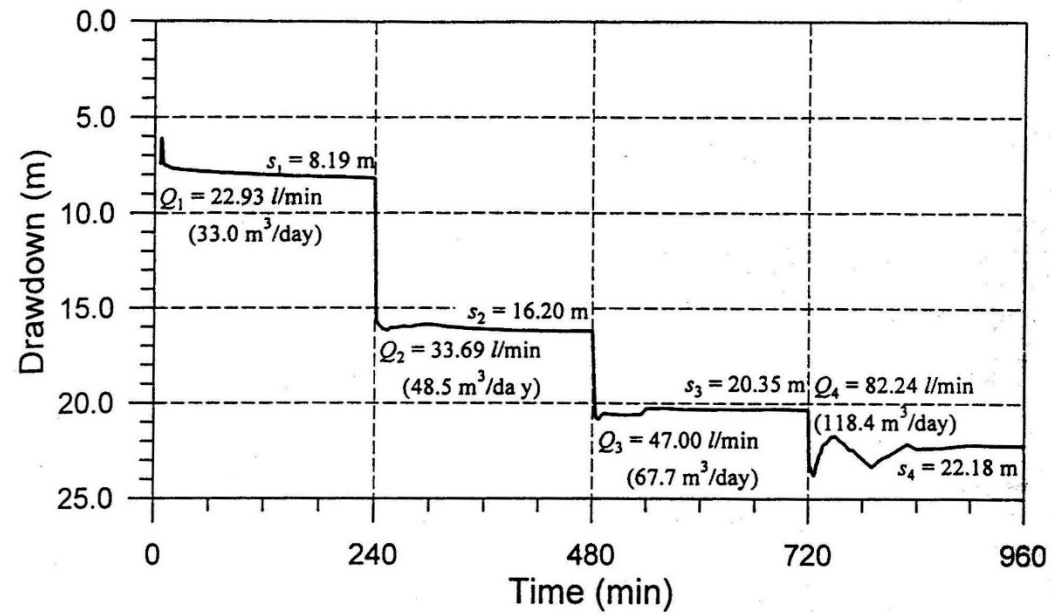
$$= \frac{1}{SC}$$

BH288



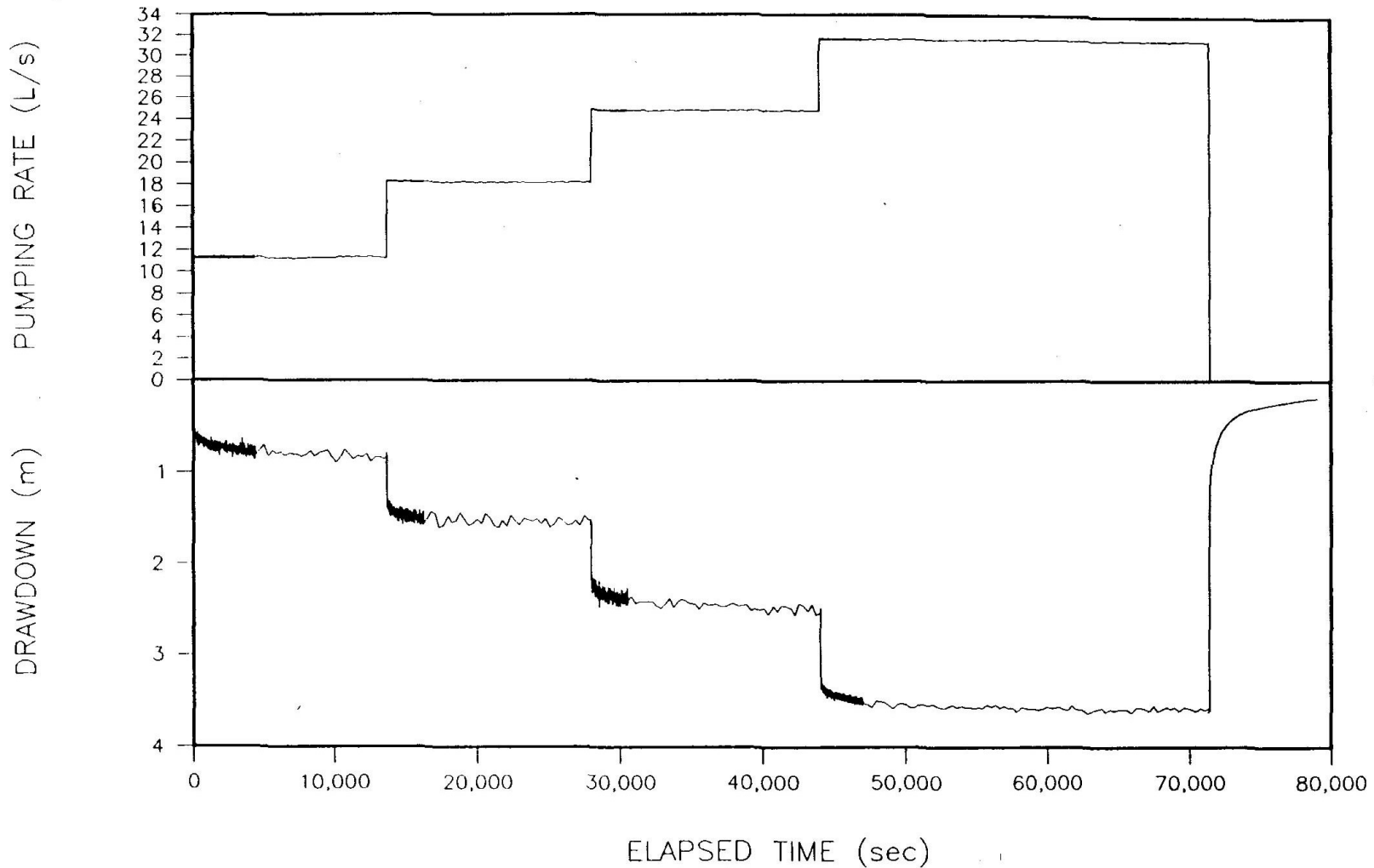
Step	Q (m^3/day)	s_w (m)	s_w/Q ($\text{m}/\text{m}^3/\text{day}$)
1	11.95	7.35	0.62
2	20.91	15.18	0.73
3	33.00	24.79	0.75
4	48.50	32.68	0.67

BH199

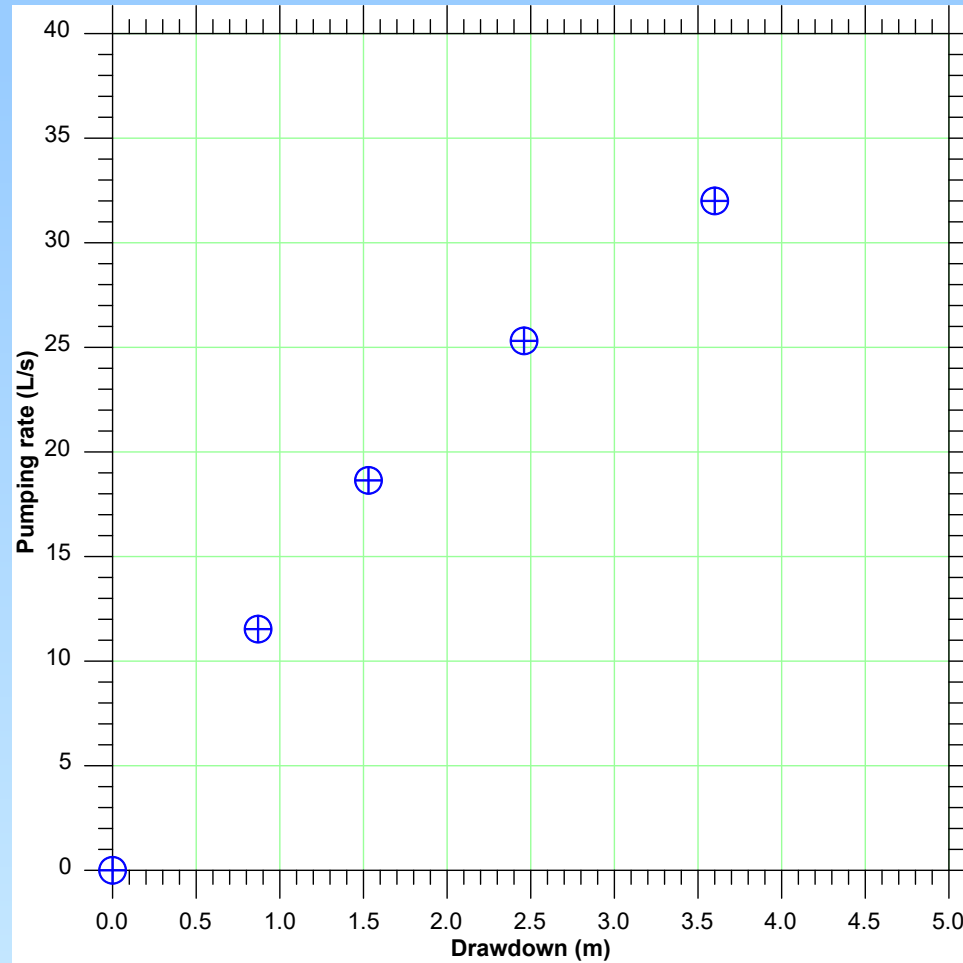


Step	Q (m^3/day)	s_w (m)	s_w/Q (m/ m^3/day)
1	33.0	8.19	0.25
2	48.5	16.20	0.33
3	67.7	20.35	0.30
4	118.4	22.18	0.19

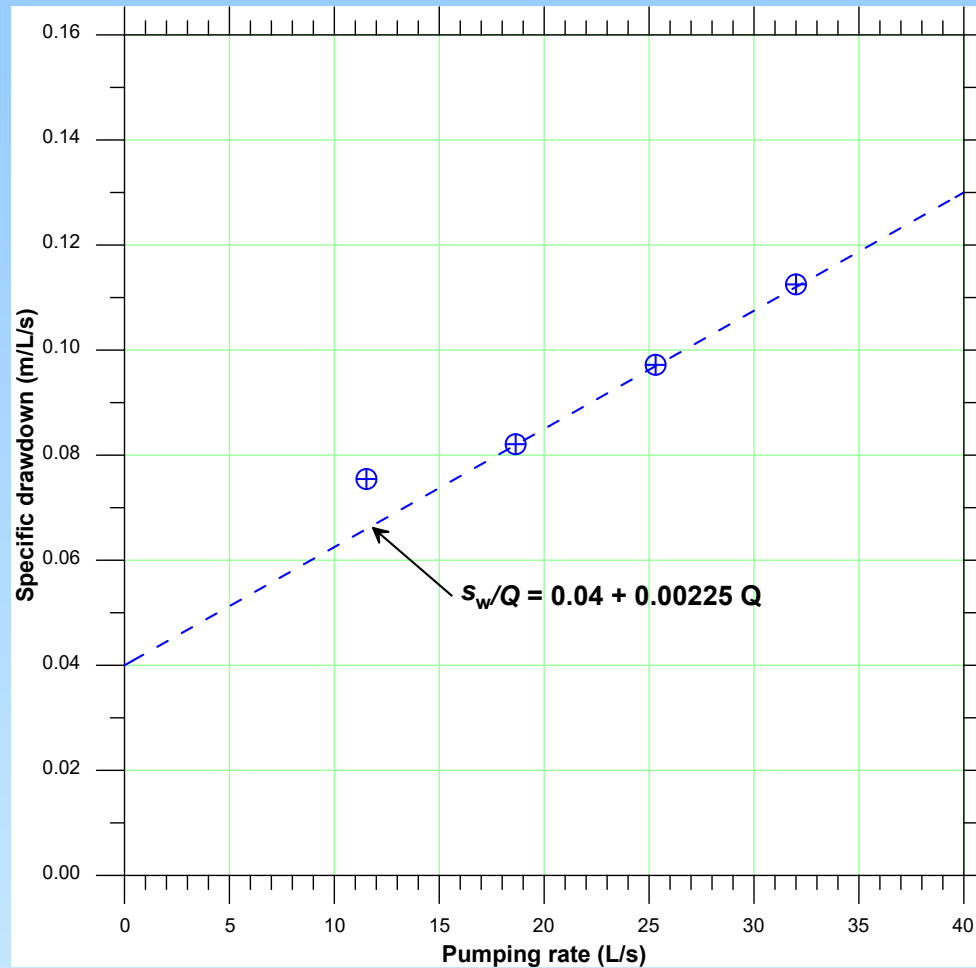
Case study: City of Guelph PW06-03



Specific capacity plot: Q vs. s_w



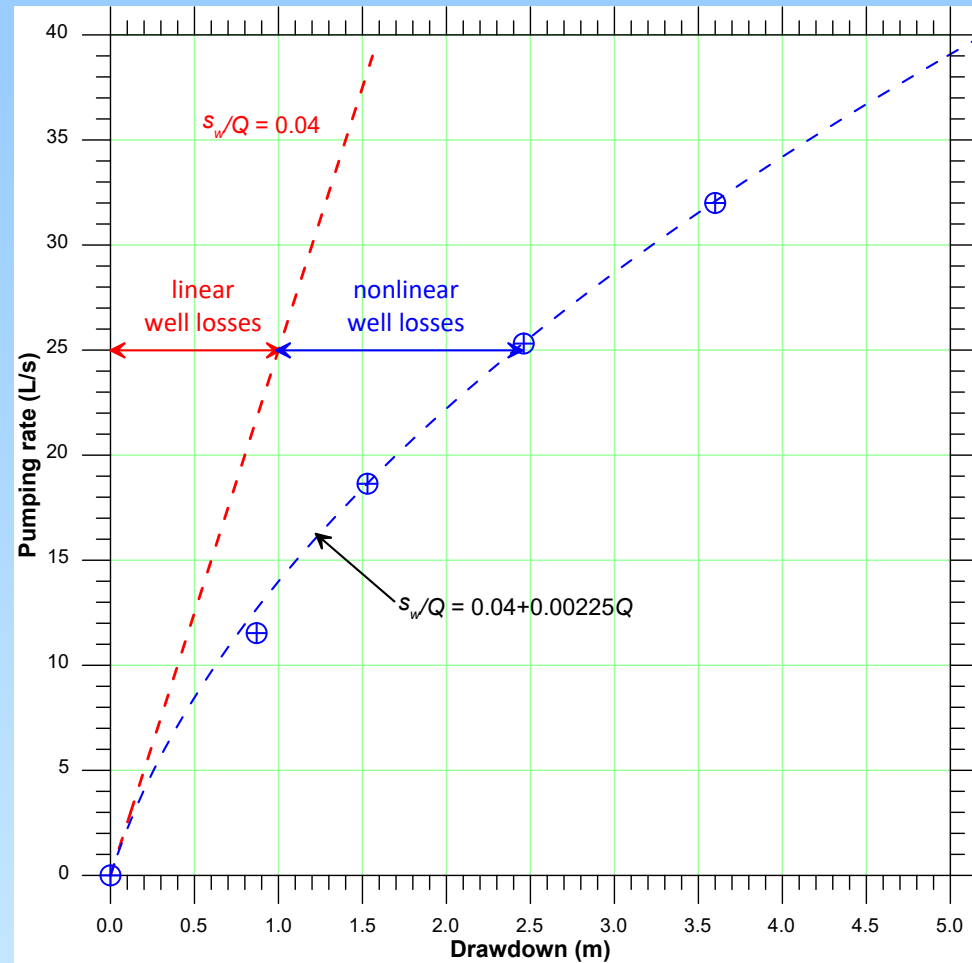
Hantush-Bierschenk plot: s_w/Q vs. Q



$$B = 0.04 \text{ m/(L/s)}$$

$$C = 2.25 \times 10^{-3} \text{ m/(L/s)}^2$$

Inference of linear and nonlinear well losses



Significance of the parameter B

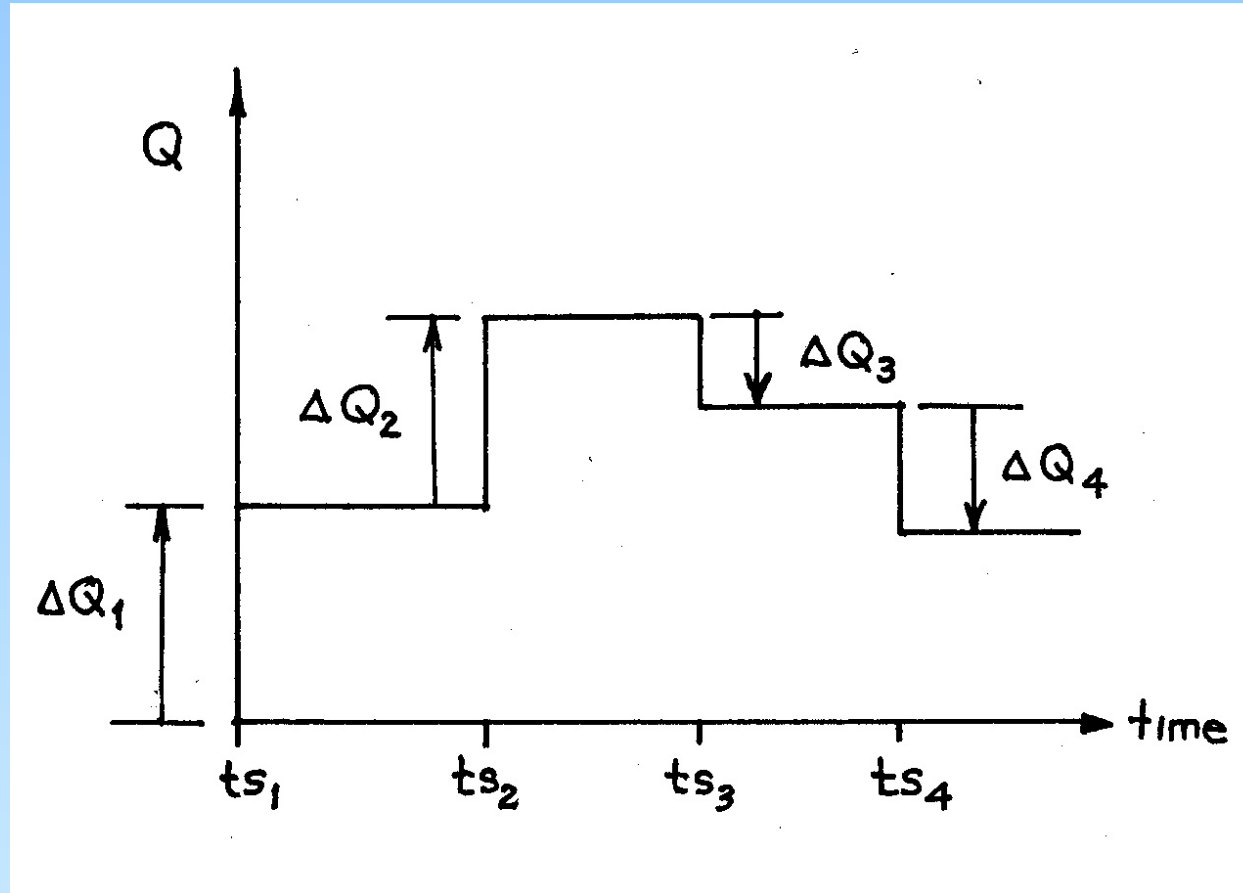
The parameter B represents the reciprocal of the specific capacity with the nonlinear well losses removed:

$$SC = \frac{1}{B} = \frac{1}{0.04 \text{ m}/(\text{L/s})} = 25 \text{ (L/s)/m}$$

$$T \sim 1.4 \times \frac{1}{B} = 1.4 \times 25 \text{ (L/s)/m} \left| \frac{86.4 \text{ m}^3/\text{d}}{\text{L/s}} \right|$$

$$= 3,020 \text{ m}^2/\text{d}$$

Interpretation of transient drawdown record: Superposition



Generalization of Theis solution

$$s_w(t) = \sum_{n=1}^{NP(t)} \frac{\Delta Q_n}{4\pi T} W \left(\frac{r_w^2 S}{4T(t - t_{s_n})} \right) \\ + \frac{Q}{4\pi T} 2S_w + CQ^P$$

Fitting the generalized Theis model

Parameters:

1. Transmissivity, T
2. Storage coefficient, S
3. Dimensionless skin factor, S_w
4. Nonlinear well loss coefficient, C
5. Well loss exponent, P

Nonuniqueness:

More than one set of parameters will match the data

Step test example

Specified parameter values

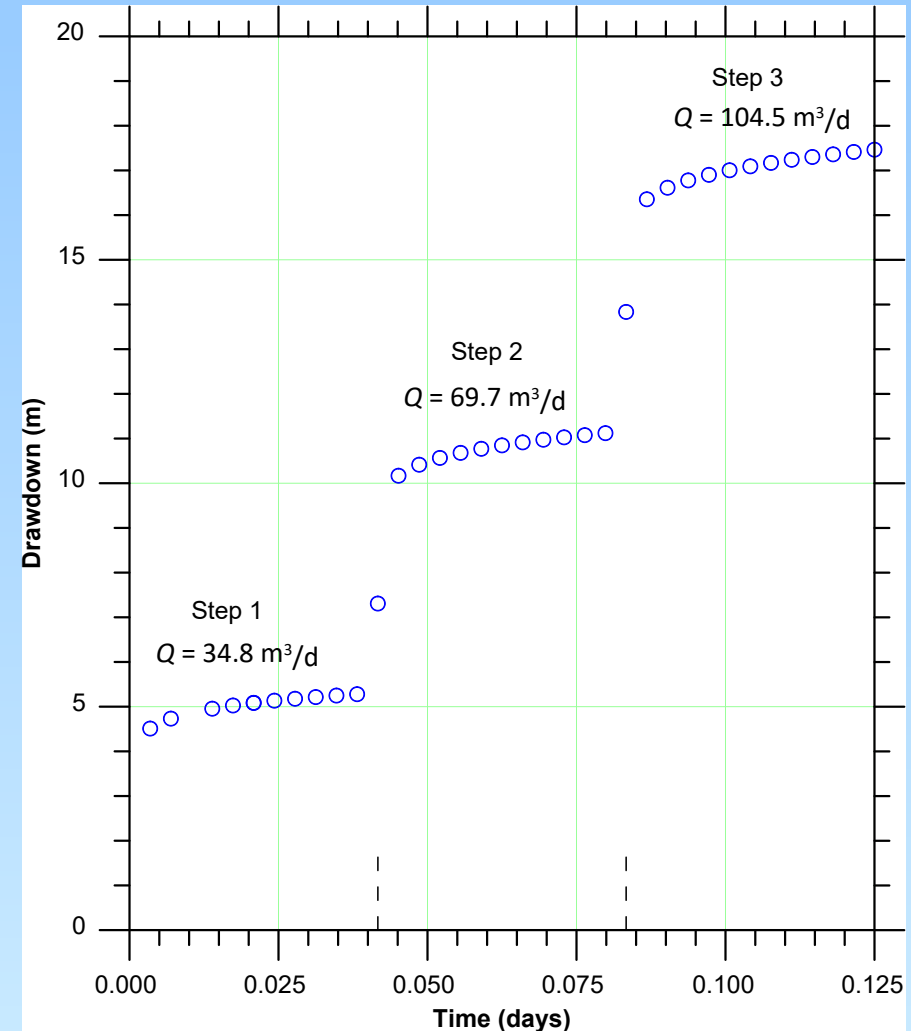
Transmissivity, T : $8.64 \text{ m}^2/\text{d}$

Storage coefficient, S : 1.0×10^{-4}

Dimensionless skin factor, S_w : 0.5193

Nonlinear well loss coefficient, C : $1.34 \times 10^{-4} \text{ m}^{-5}\text{d}^2$

Well loss exponent, P : 2.0



Automatic fit

Fitted parameter values

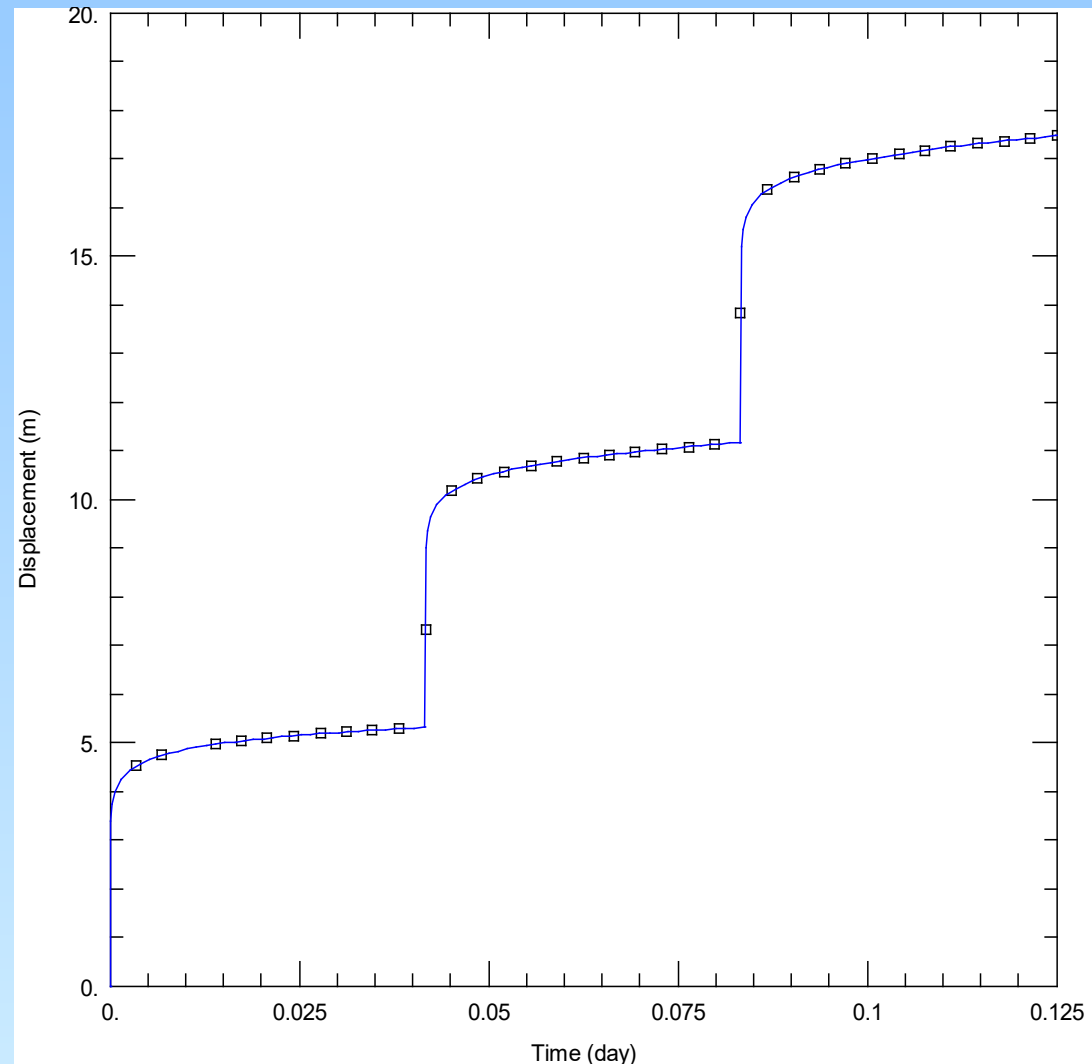
Transmissivity, T : 8.65 m²/d

Storage coefficient, S : 5.55×10^{-5}

Dimensionless skin factor, S_w : 0.2379

Nonlinear well loss coefficient, C : 1.33×10^{-4} m⁻⁵d²

Well loss exponent, P : 2.0



Specified vs. fitted parameter values

Parameter	Specified value	Fitted value
Transmissivity, T	8.64 m ² /d	8.65 m ² /d
Storage coefficient, S	1.0×10^{-4}	5.55×10^{-5}
Dimensionless skin factor, S_w	0.52	0.24
Nonlinear well loss coefficient, C	$1.34 \times 10^{-4} \text{ m}^{-5}\text{d}^2$	$1.33 \times 10^{-4} \text{ m}^{-5}\text{d}^2$
Well loss exponent, P	2.0	2.0

Nonuniqueness:

Multiple parameter sets yield the same match

DIAGNOSTICS REPORT

Diagnostic Statistics

Estimation complete! Parameter change criterion (ETOL) reached.

Aquifer Model: Confined
Solution Method: Theis (Step Test)

Estimated Parameters

Parameter	Estimate	Std. Error	Approx. C.I.	t-Ratio	
T	8.649	0.004118	+/- 0.008397	2100.2	m ² /day
S	5.554E-5	4.83E-5	+/- 9.848E-5	1.15	
Sw	0.2379	0.4298	+/- 0.8763	0.5536	
C	0.0001325	1.319E-7	+/- 2.689E-7	1004.7	day ² /m ⁵
P	2.	not estimated			

C.I. is approximate 95% confidence interval for parameter
t-ratio = estimate/std. error
No estimation window

$K = T/b = 0.8649$ m/day (0.001001 cm/sec)
 $Ss = S/b = 5.554E-6$ 1/m

Parameter Correlations

	T	S	Sw	C
T	1.00	-0.85	-0.85	0.31
S	-0.85	1.00	1.00	-0.21
Sw	-0.85	1.00	1.00	-0.21
C	0.31	-0.21	-0.21	1.00

The storage coefficient and skin factor are perfectly correlated.

Residual Statistics

for weighted residuals

Sum of Squares 4.392E-5 m²
Variance 1.417E-6 m²
Std. Deviation 0.00119 m
Mean -0.0001331 m
No. of Residuals 35
No. of Estimates 4

Why are the storage coefficient (S) and the skin factor (S_w) perfectly correlated?

Let's go back to the Cooper-Jacob solution, with additional skin losses:

$$s_w \cong \frac{Q}{4\pi T} \left[\left(-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right) + 2S_w \right]$$

We can expand this as:

$$s_w = \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \text{EXP}\{-2S_w\} \right\} \right]$$

In other words, we obtain the same drawdown from:

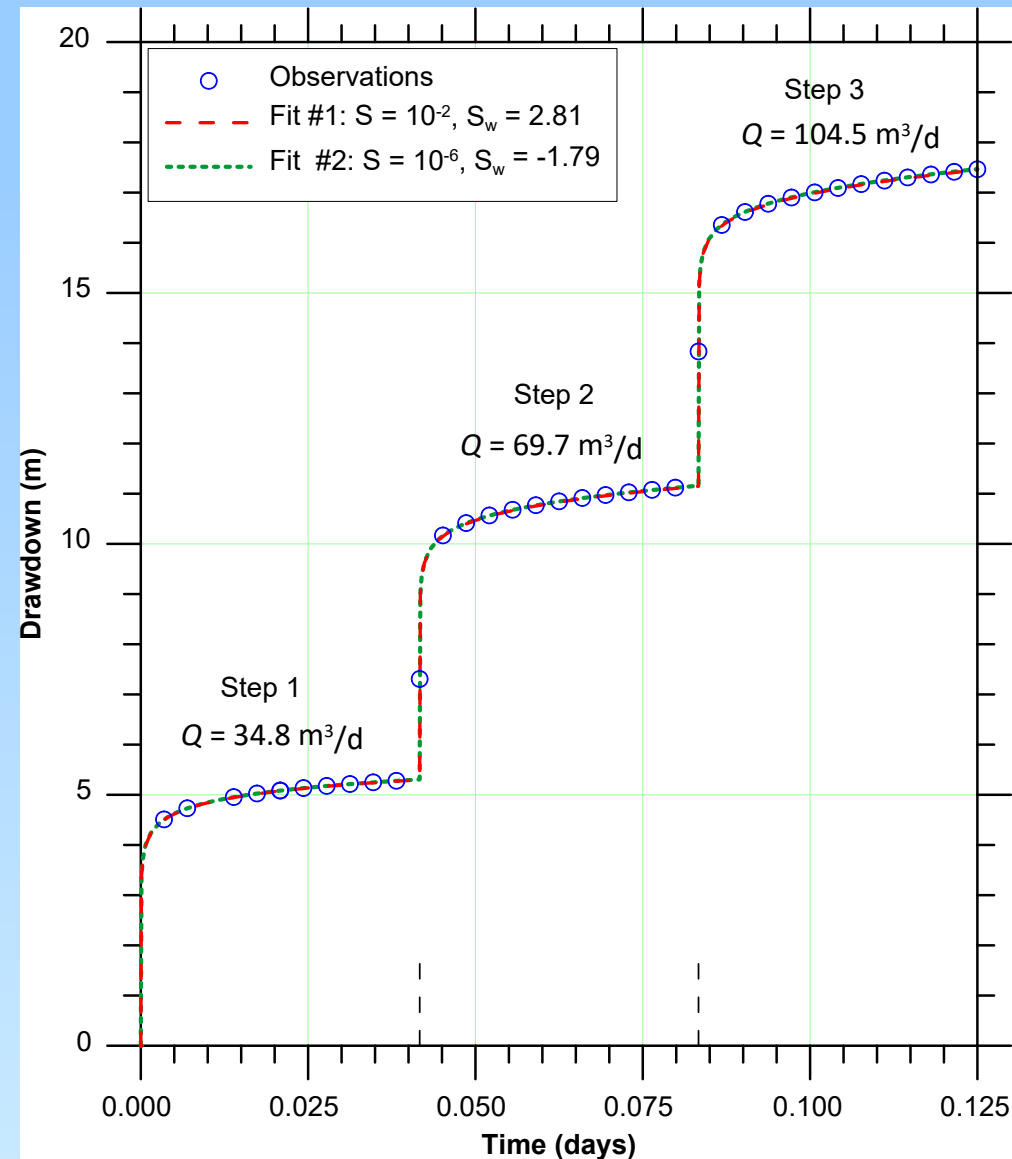
$$s_w = \frac{Q}{4\pi T} \left[\left(-0.5772 - \ln \left\{ \frac{r_w^2 S}{4Tt} \right\} \right) + 2s_w \right]$$

as we do from:

$$s_w = \frac{Q}{4\pi T} \left[-0.5772 - \ln \left\{ \frac{r_w^2 S_E}{4Tt} \right\} \right]$$

with: $S_E = S \text{ EXP}\{-2s_w\}$

Step test: Two alternative fits



Results of multiple fits

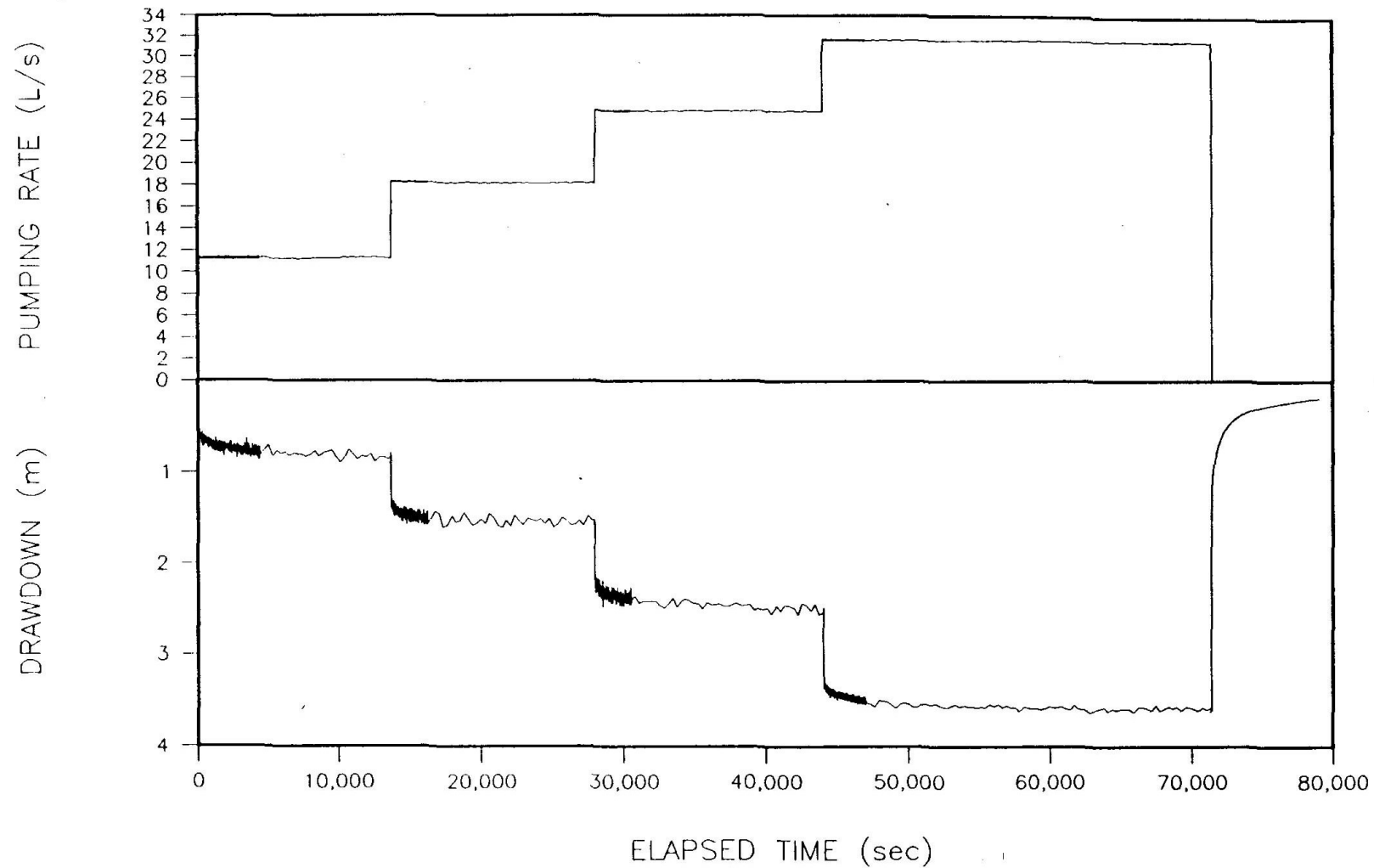
Fixed S	Fitted S_w	$S \text{ EXP}\{-2S_w\}$
1×10^{-6}	-1.785	3.55×10^{-5}
1×10^{-5}	-0.634	3.55×10^{-5}
1×10^{-4}	0.517	3.55×10^{-5}
1×10^{-3}	1.667	3.55×10^{-5}
1×10^{-2}	2.810	3.63×10^{-5}

We cannot estimate the storage coefficient and skin factor separately.

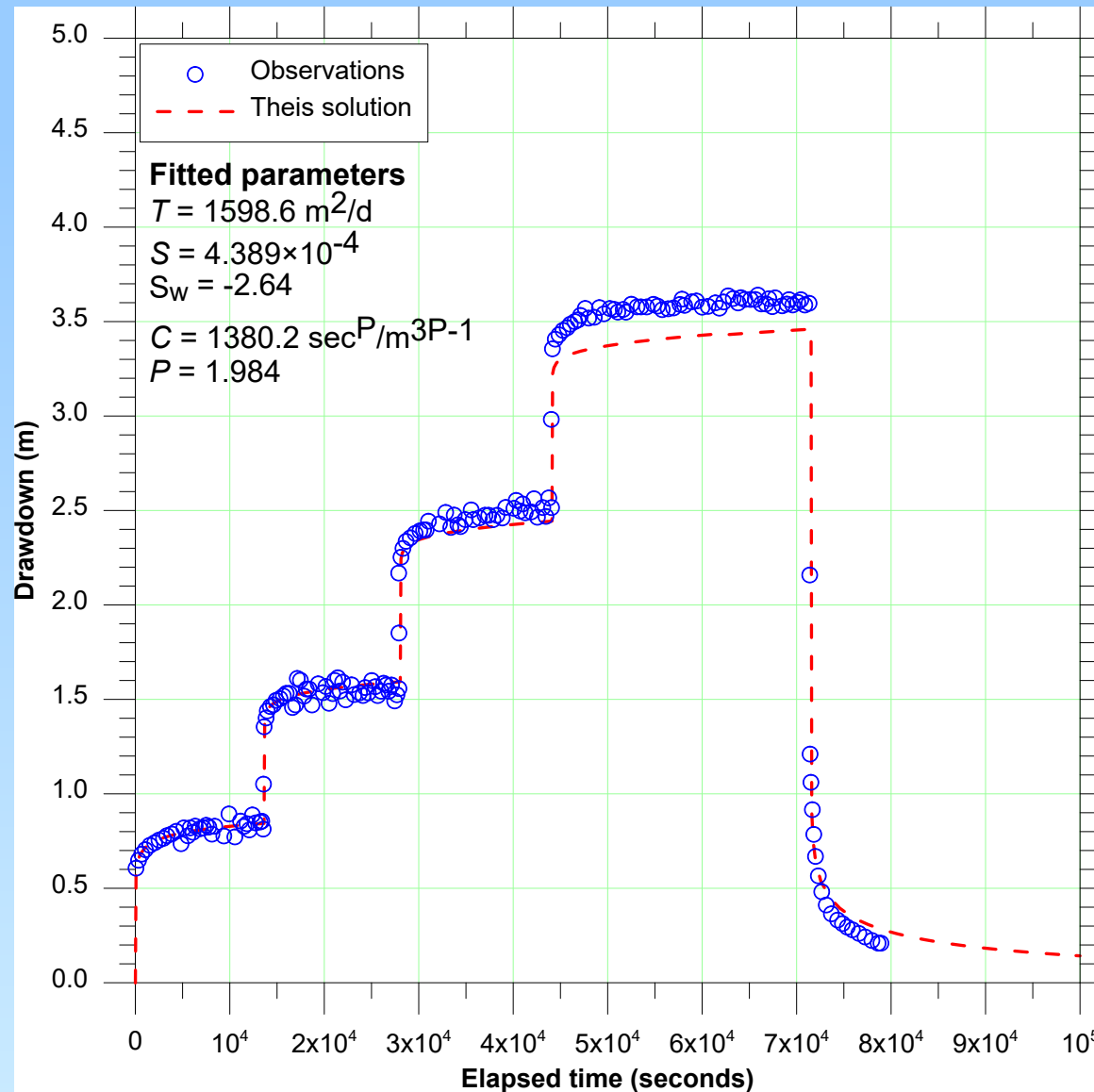
CJN preferred analysis approach: “Constrained” fitting

1. Fix S at realistic value
2. Neglect skin losses [unless there is a good reason to include them], $S_w = 0.0$
3. Fix C from Hantush-Bierschenk analysis
4. Fix $P = 2.0$
5. Assess the significance of S with respect to the estimation of transmissivity

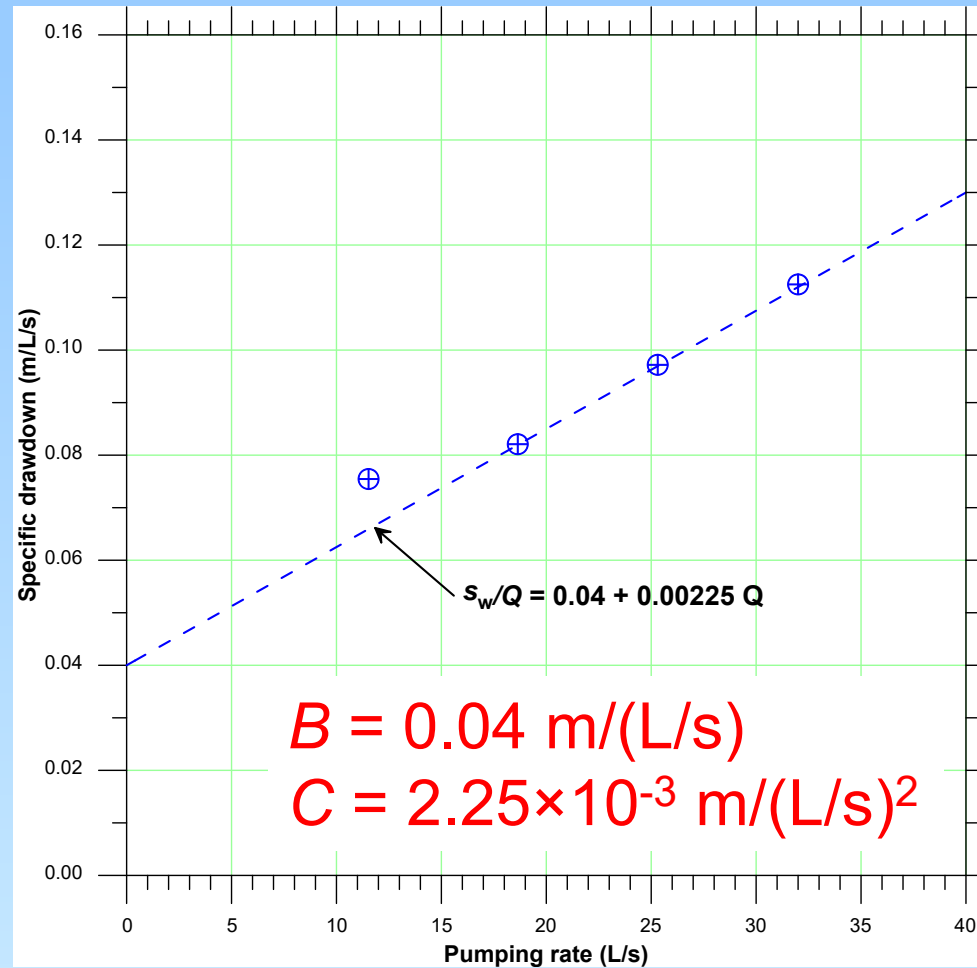
City of Guelph PW06-03 (2)



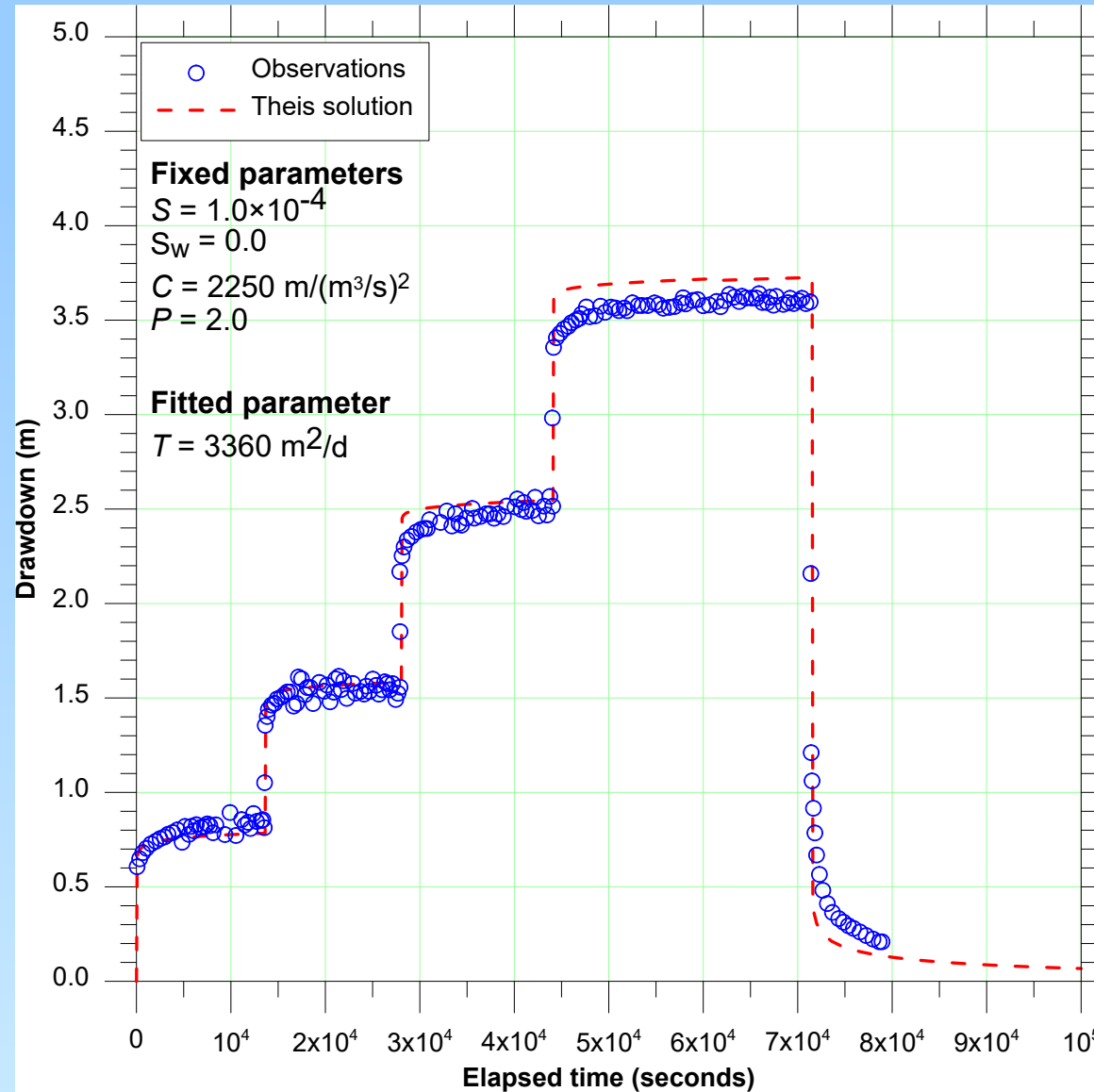
Analysis #1: Automatic with all parameters adjustable



Hantush-Bierschenk plot

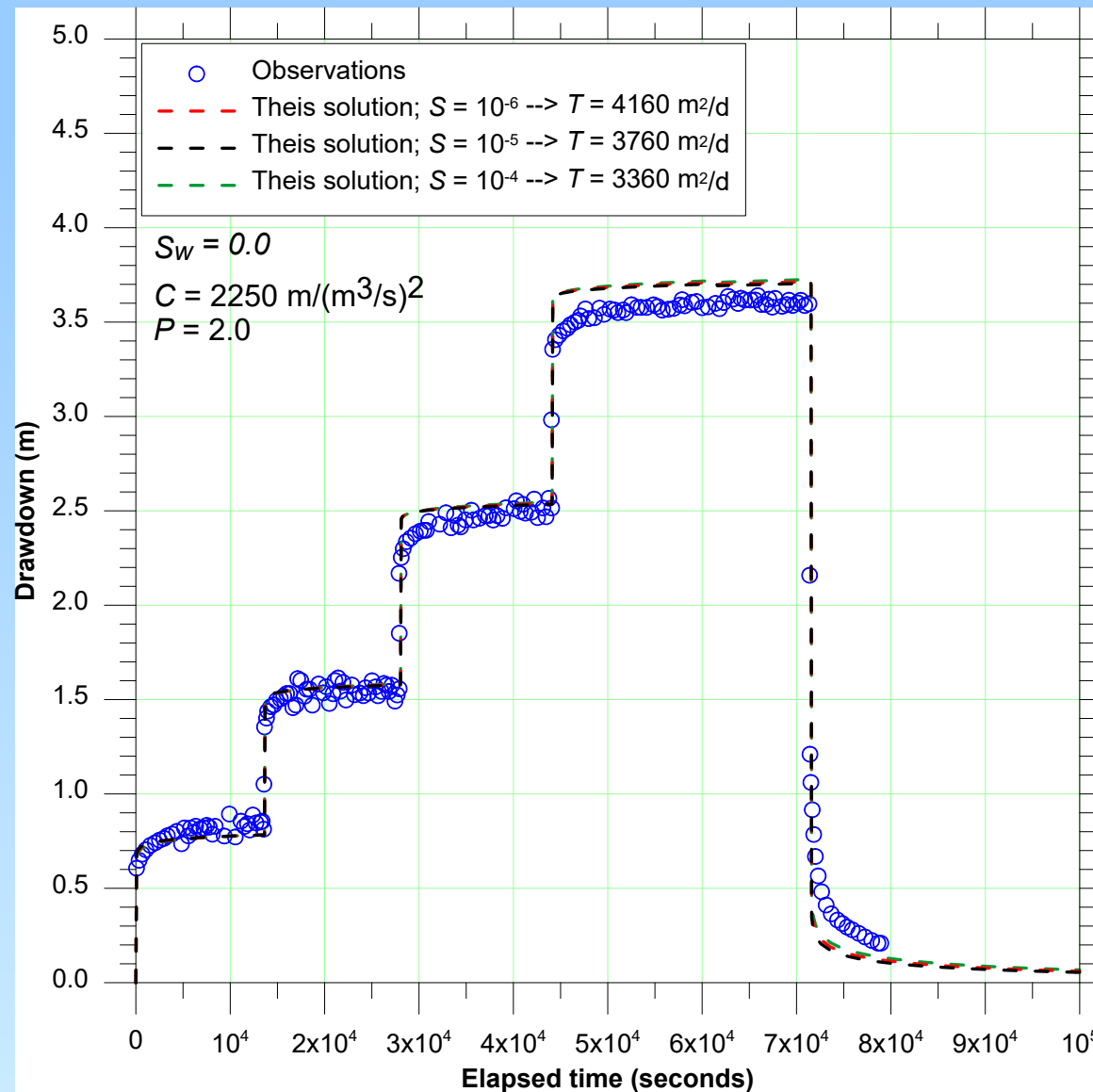


Analysis #2: “Constrained fit”



T specific capacity = 3,020 m^2/d

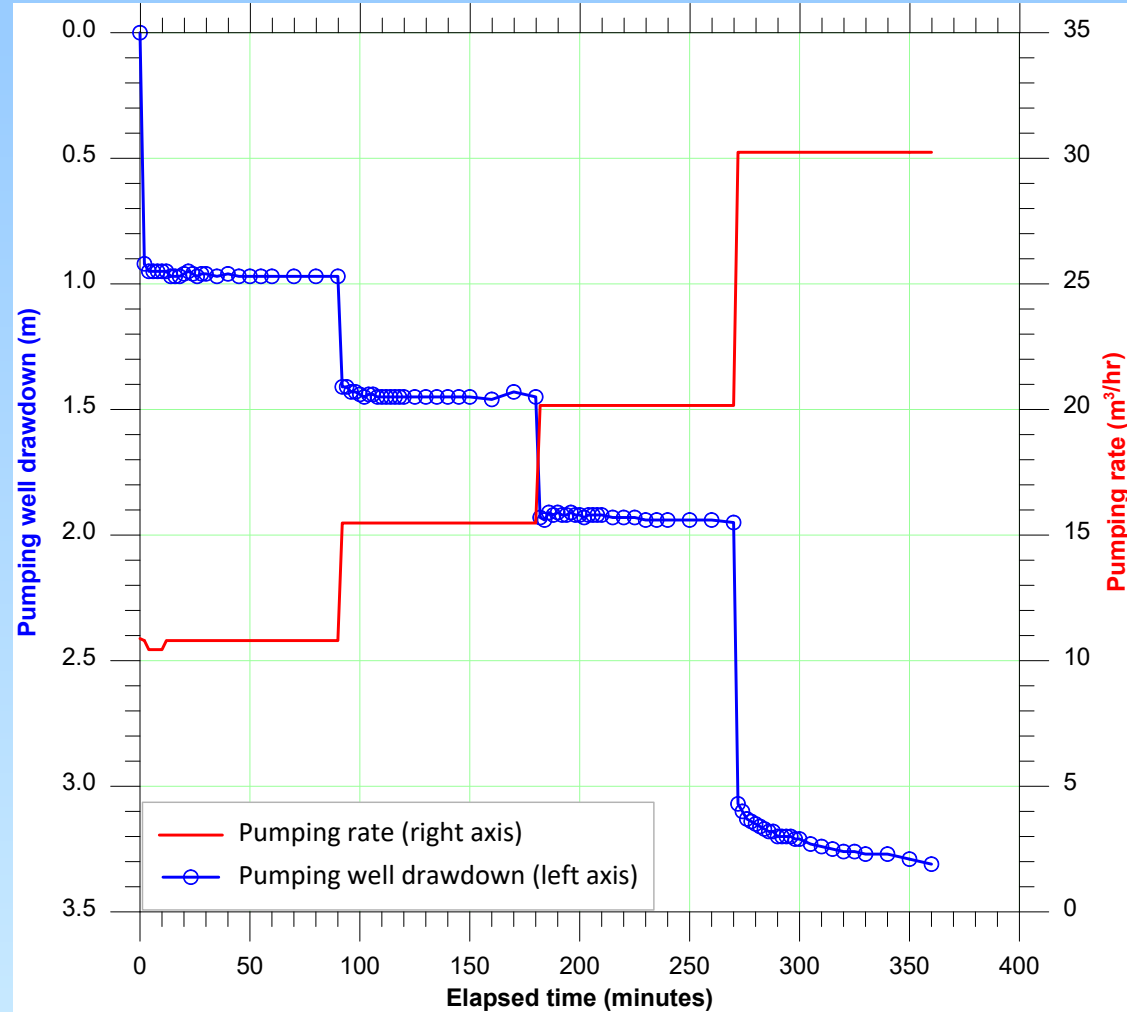
Significance of the storativity S



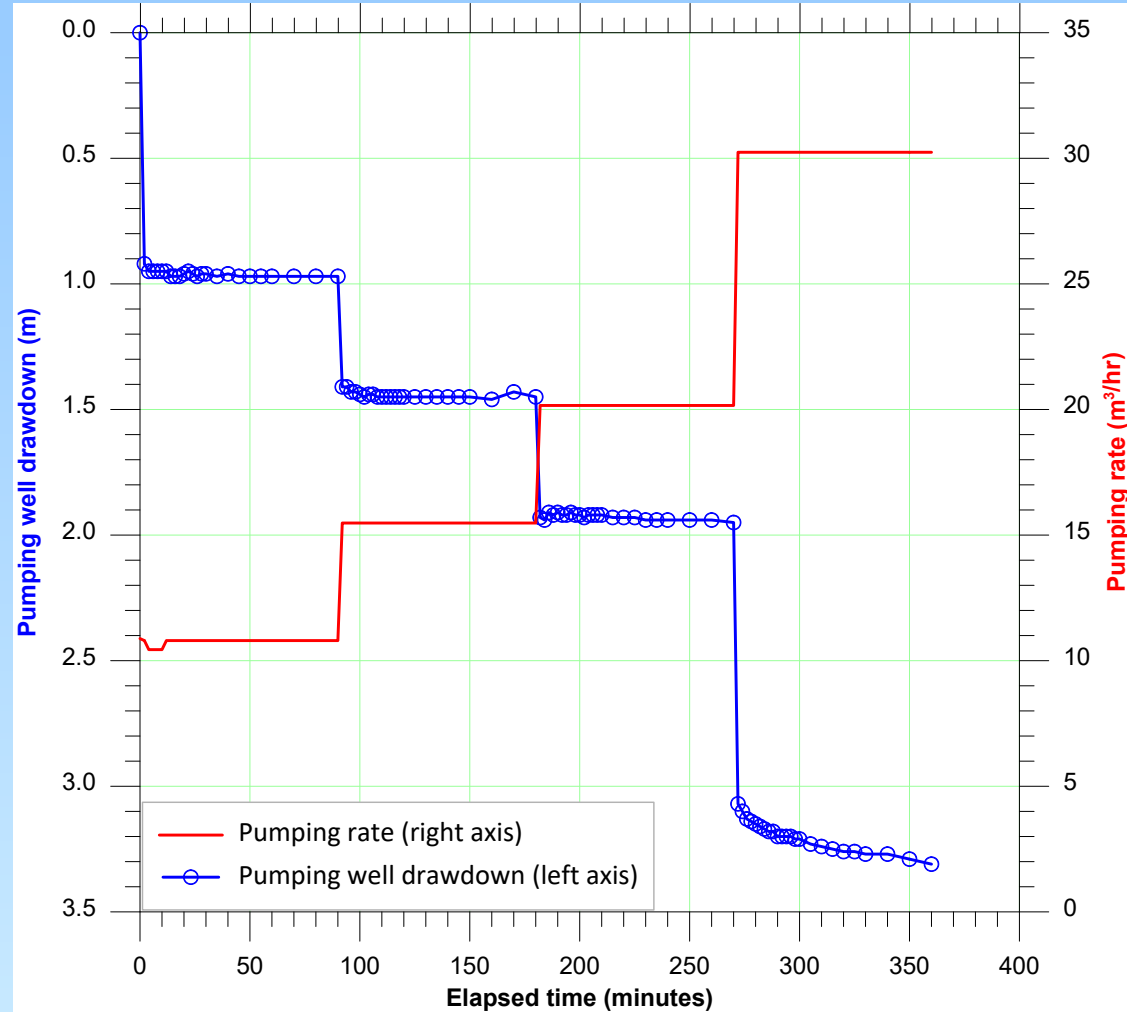
Internally consistent analysis of step test and constant-rate test

Case study:
Kandieng Reay, Cambodia
Well BHKR-1

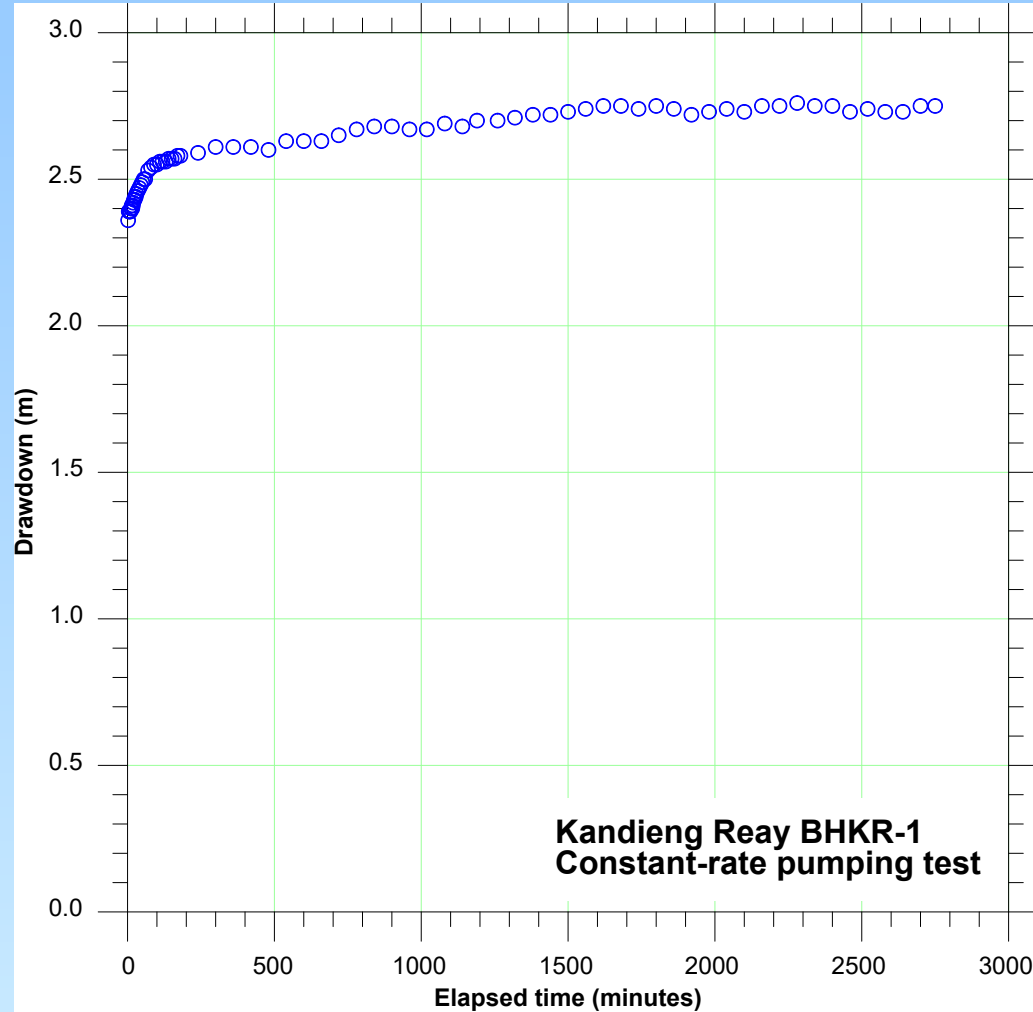
1. Step test



1. Step test

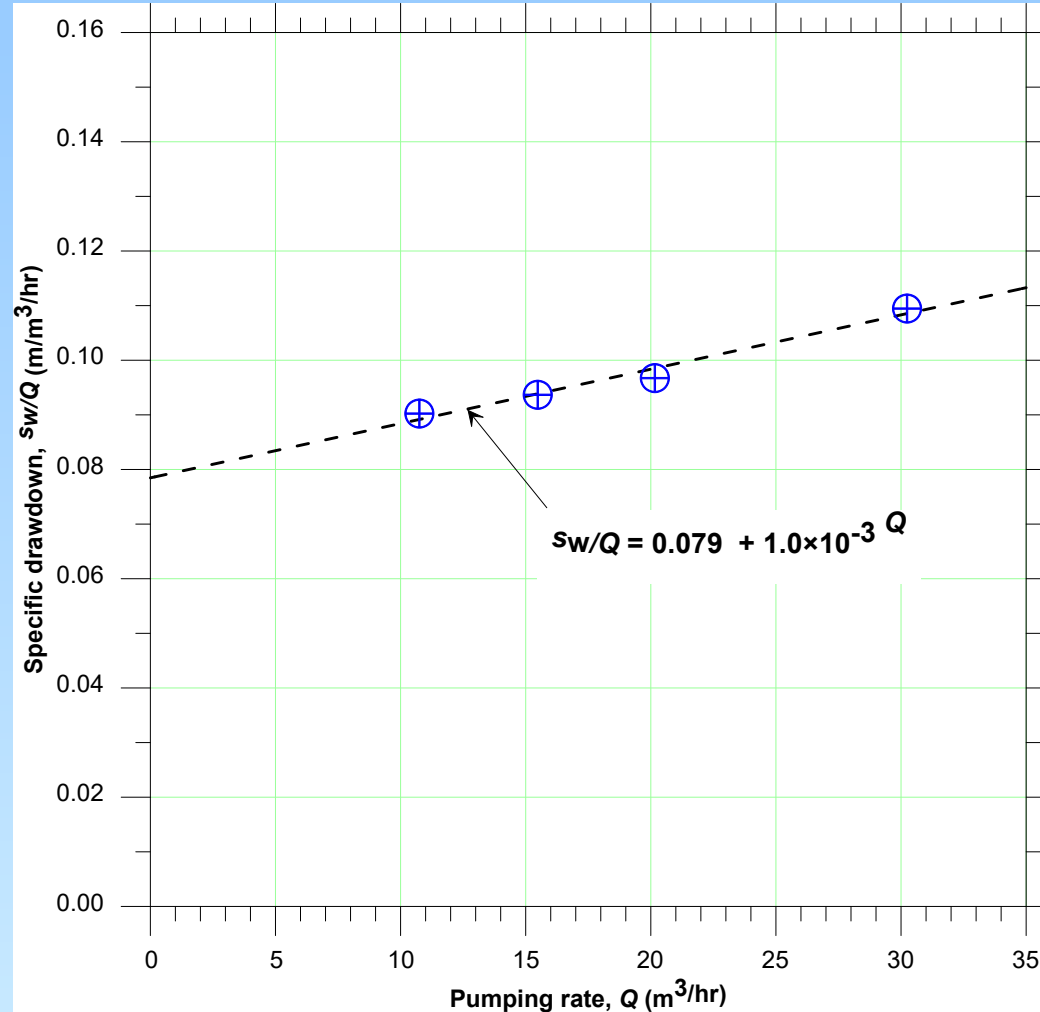


2. Constant-rate test



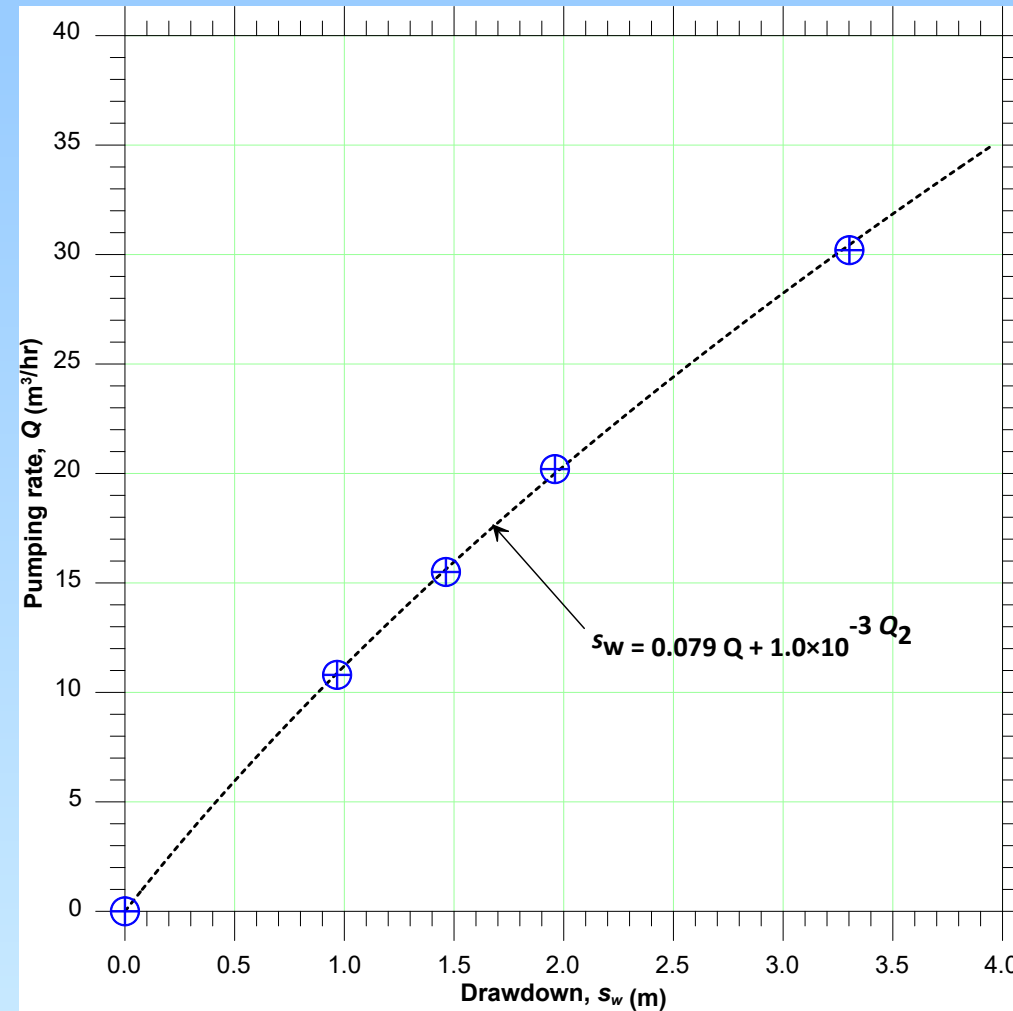
46 hours of pumping
 $Q_{\text{avg}} = 25.2 \text{ m}^3/\text{hr}$

Estimation of nonlinear well losses

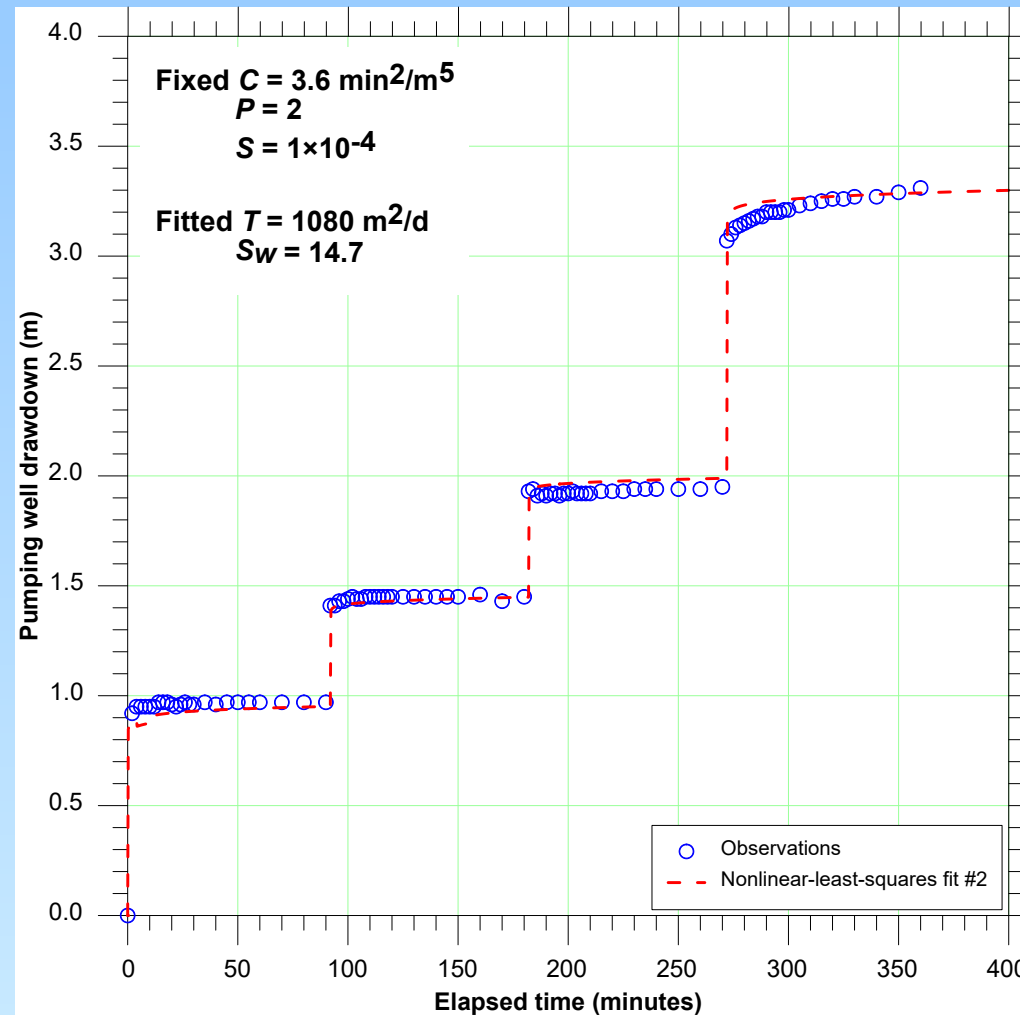


Hantush-Bierschenk plot:
 $C = 1.0 \times 10^{-3} \text{ m}/(\text{m}^3/\text{hr})^2$

Check on the Hantush-Bierschenk analysis

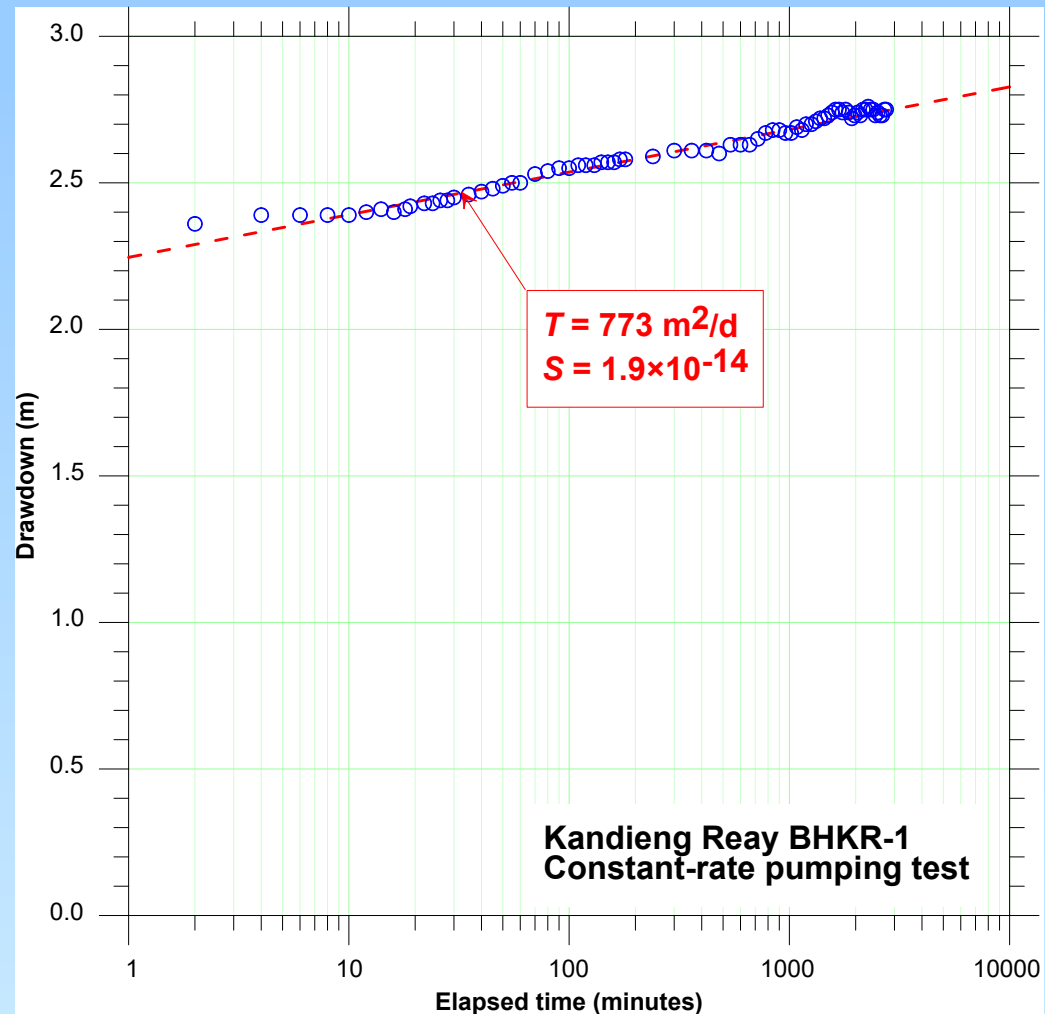


Constrained step test analysis



Conversion:
 $C = 1.0 \times 10^{-3} \text{ m}/(\text{m}^3/\text{hr})^2$
 $= 3.6 \text{ m}/(\text{m}^3/\text{min})^2$

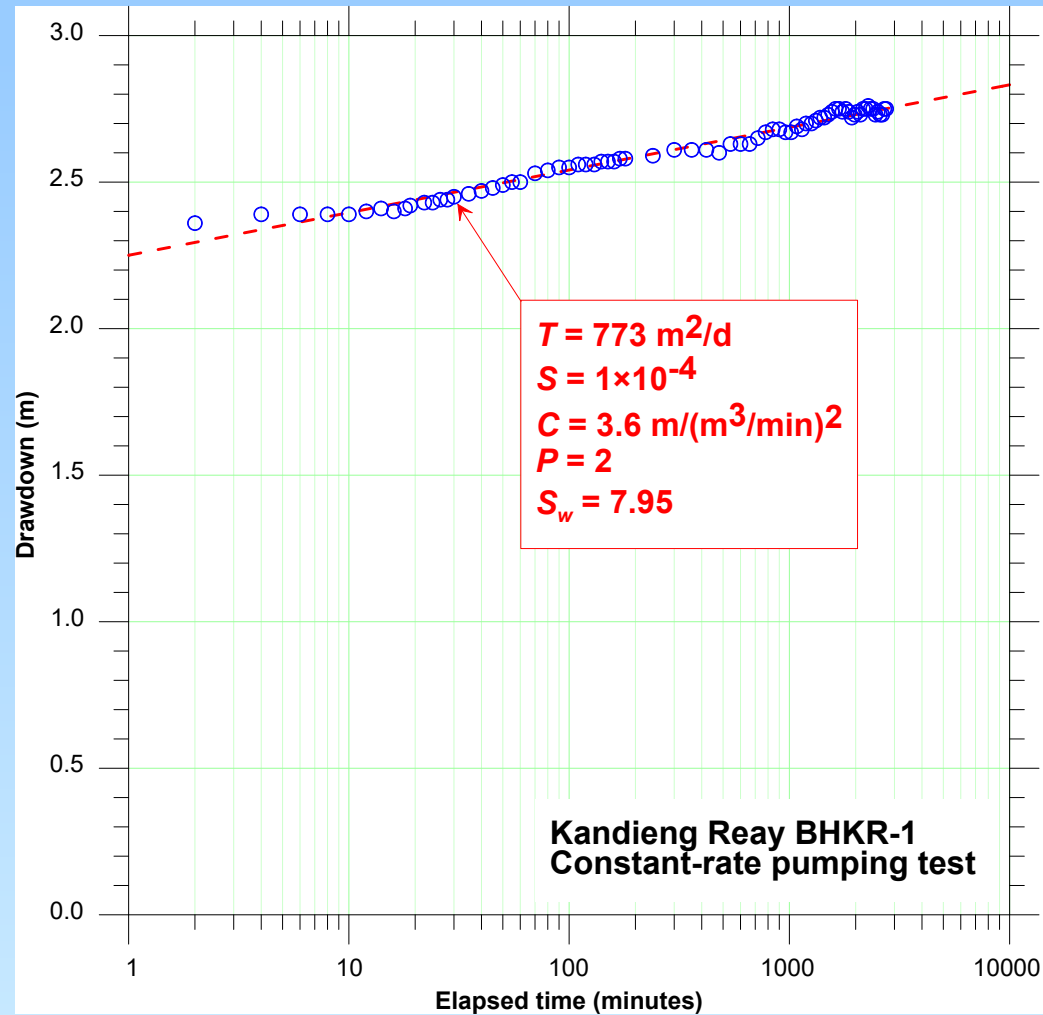
Estimation of the transmissivity



Re: transmissivity
estimate from the step test
analysis: $1080 \text{ m}^2/\text{day}$

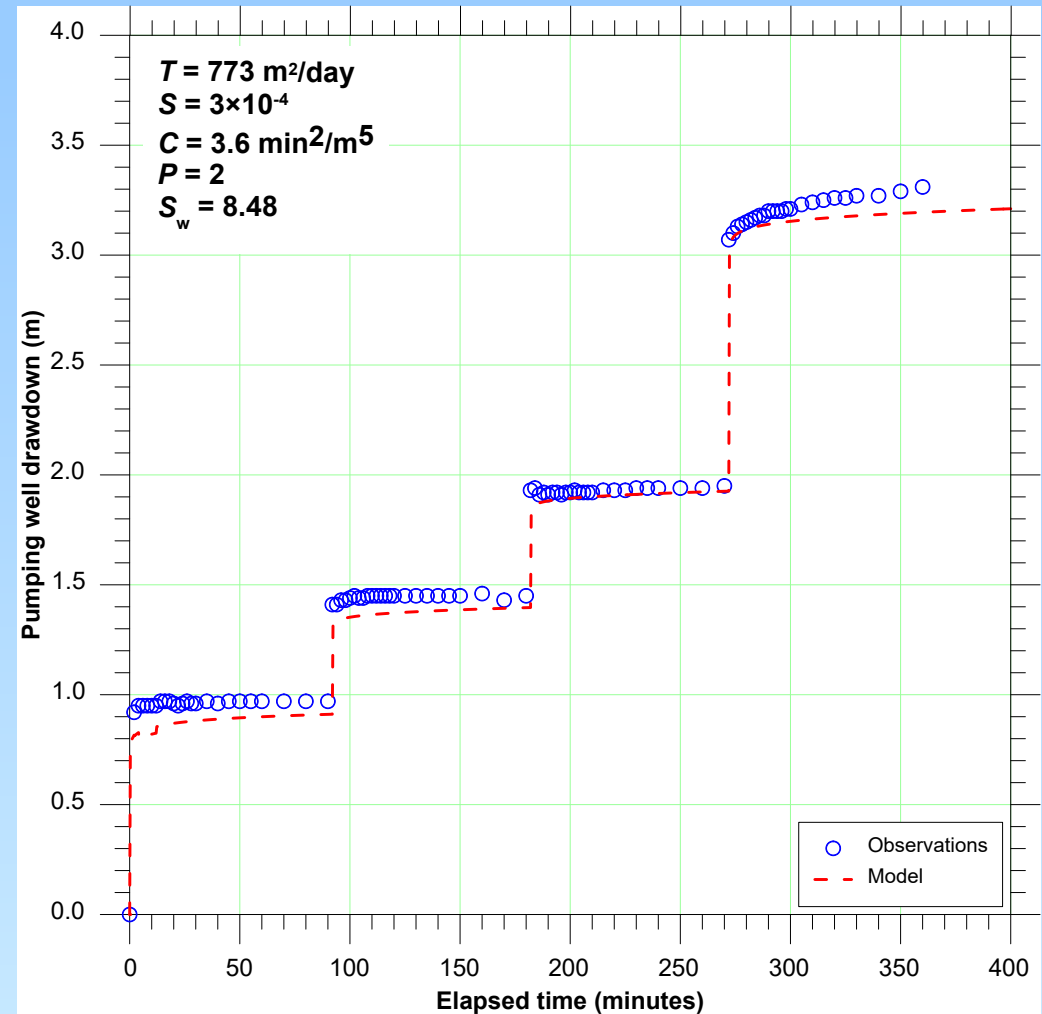
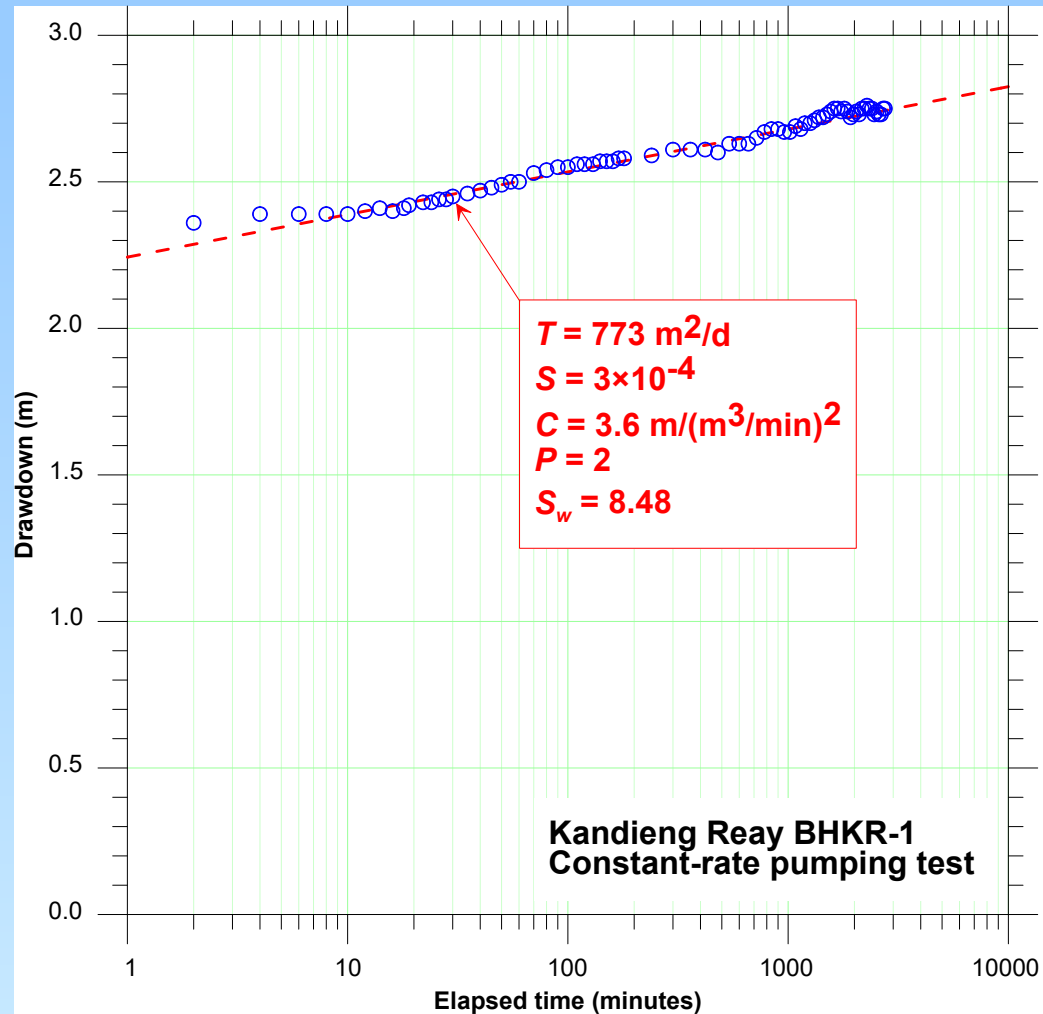
Note the non-physical
storativity estimate

Cooper-Jacob analysis #2

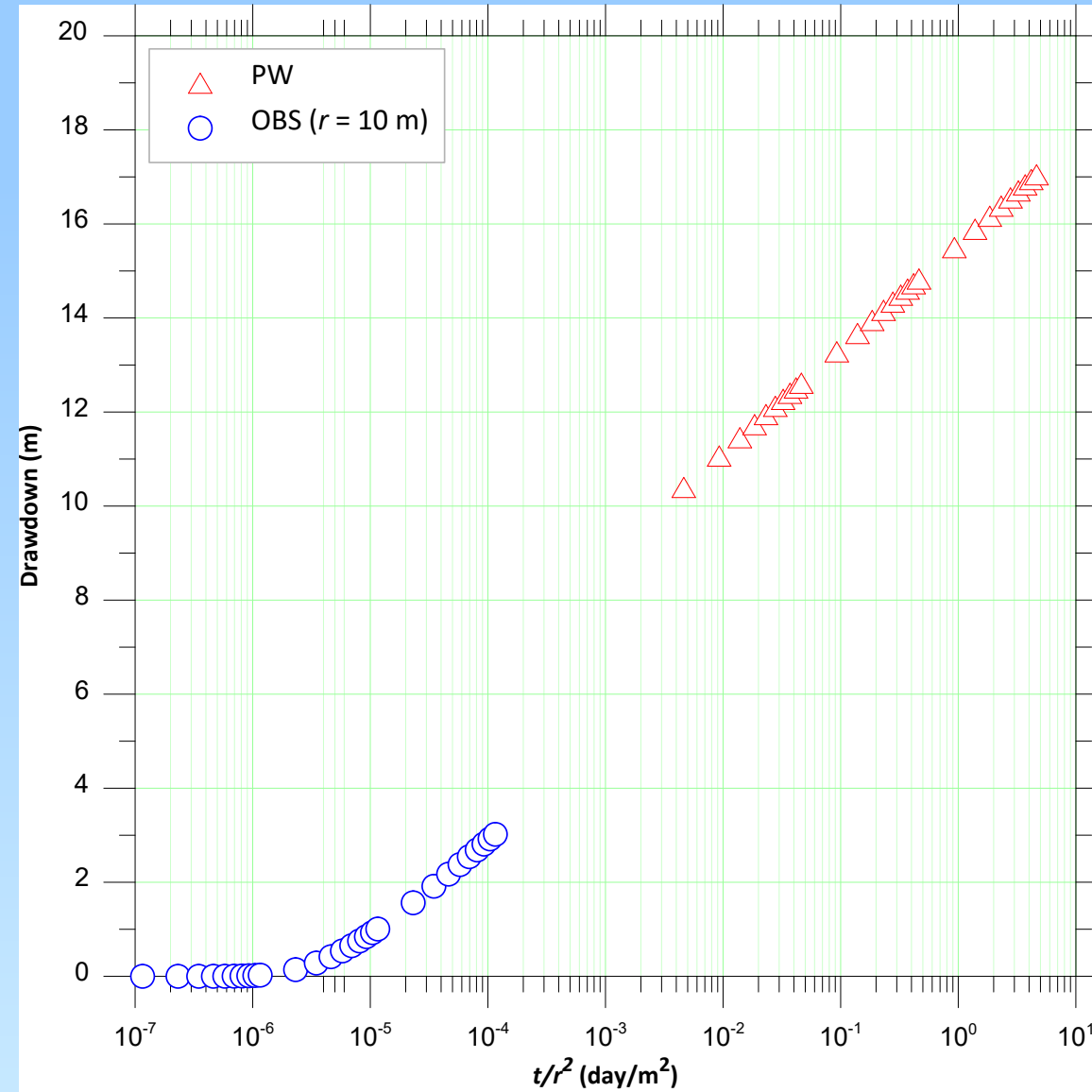


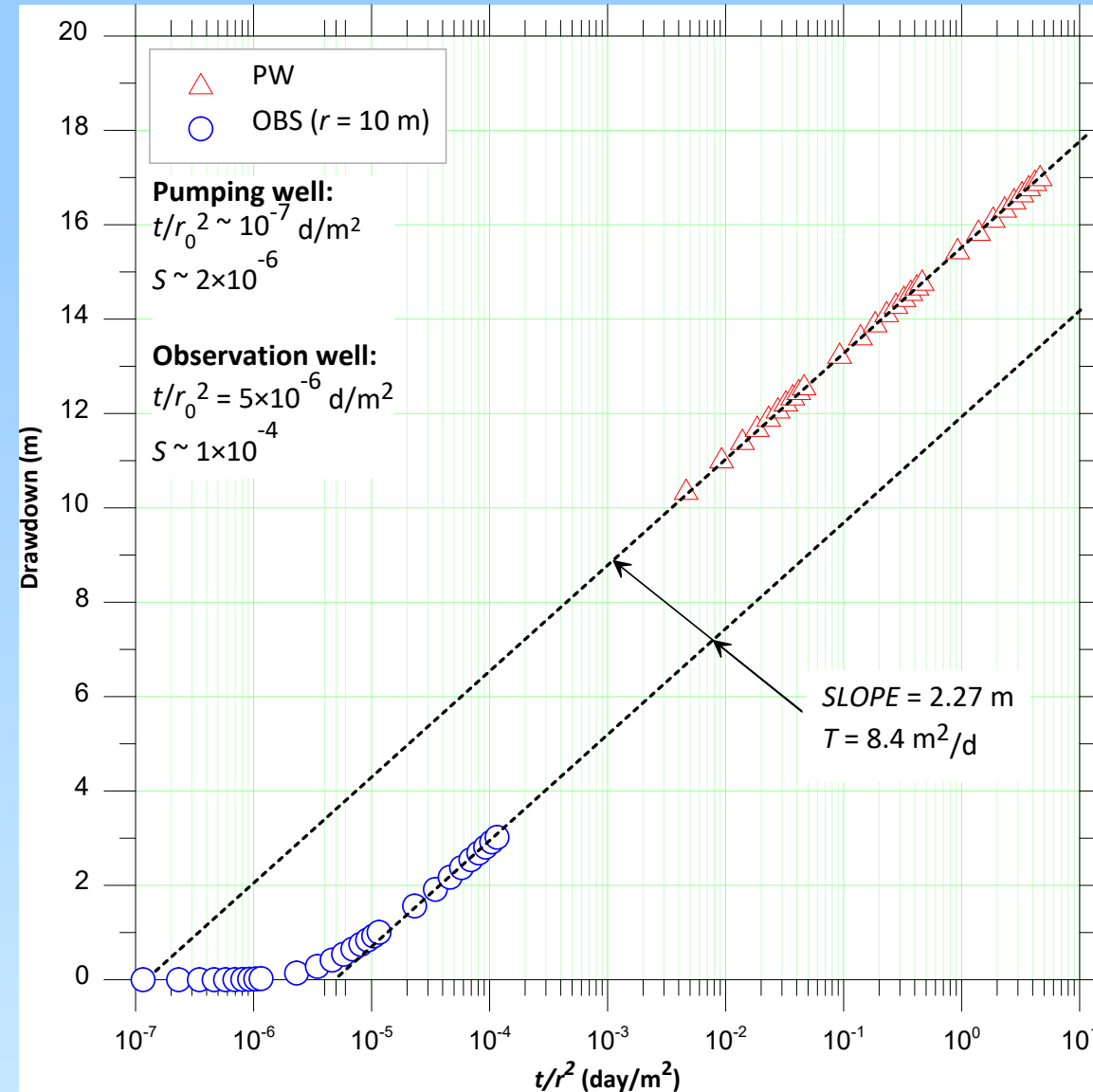
The storativity is fixed to estimate the dimensionless skin factor.

One final iteration



Synthesis of pumping well and observation well drawdowns





Take-Home Points (1)

1. Head loss in the formation is only one of the components of the drawdown in a pumping well. An understanding of the sources of drawdown provides a firm basis for accounting for well performance.
2. Cooper-Jacob straight-line analyses provide the most representative estimates of transmissivity.

Take-Home Points (2)

4. Step tests are the only definitive method of evaluating non-aquifer well losses.
5. We usually cannot estimate all parameter values. Watch for non-physical parameter estimates and parameter correlation.
6. Where the data are available, we should try to obtain a consistent set of parameters from step and constant-rate pumping tests.